Heat & Thermodynamics and Gases

Fill Ups

Q.1. One mole of a mono-atomic ideal gas is mixed with one mole of a diatomic ideal gas. The molar specific heat of the mixture at constant volume is ....

Ans. 4 cal.

Solution.

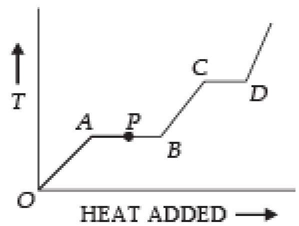
$$\bar{C}_v = \frac{n_1 C_{v_1} + n_2 C_{v_2}}{n_1 + n_2}$$

$$= \frac{1 \times \frac{3}{2} R + 1 \times \frac{5}{2} R}{1+1} = 2R$$

Q.2. The variation of temperature of a material as heat is given to it at a constant rate is shown in the figure. The material is in solid state at the point O. The state of the material at the point P is ....

Ans. Partly solid and partly liquid

Solution.



Q.3. During an experiment, an ideal gas is found to obey an additional law VP^2 =constant, The gas is initially at a temperature T, and volume V. When it expands to a volume 2V, the temperature becomes..........

Ans. $\sqrt{2}T$

Solution. PV = RT (Ideal gas equation)

$$\Rightarrow P = \frac{RT}{V} \quad \text{...(i)}$$

Given that VP^2 = const ... (ii)

From (i) and (ii)

$$\therefore \quad \frac{T^2}{V} = \text{const.}$$

$$\therefore \quad \frac{T_1^2}{V_1} = \frac{T_2^2}{V_2} \Rightarrow T_2 = T_1\sqrt{\frac{V_2}{V_1}} = T\sqrt{\frac{2V}{V}} = \sqrt{2}T$$

Q.4. 300 grams of water at 25° C is added to 100 grams of ice at 0°C. The final temperature of the mixture is .....°C.

Ans. 0°C

Solution. The heat required for 100 g of ice at 0° C to change into water at 0°C = mL = 100 × 80 ×4.2 = 33,600 J ... (i)

The heat released by 300g of water at 25°C to change its temperature to 0°C = mcΔT = 300 × 4.2 × 25 = 31,500 J ... (ii)

Since the energy in eq. (ii) is less than of eq. (i) therefore the final temperature will be 0°C.

Q.5. The earth receives at its surface radiation from the sun at the rate of 1400 W m^{-2}. The distance of the centre of the sun from the surface of the earth is 1.5 × 10^{11} m and the radius of the sun is 7 × 10^8 m. Treating the sun as a black body, it follows from the above data that its surface temperature is...........K.

Ans. 5803K

Solution. The energy received per second per unit area from Sun at a distance of 1.5 × 10^{11} m is 1400 J/sm^2. The total energy released by Sun/per second.

$= 1400 \times 4\pi \times (1.5 \times 10^{11})^2$.

∴ The total energy released per second per unit surface area of the Sun

$$= \frac{1400 \times 4\pi \times (1.5 \times 10^{11})^2}{4\pi \times (7 \times 10^8)^2}$$

This energy E is also equal to E = σT^4

$$\therefore \quad T = \left[\frac{1400 \times 4\pi \times (1.5 \times 10^{11})^2}{4\pi \times (7 \times 10^8)^2 \times 5.67 \times 10^{-8}}\right]^{\frac{1}{4}} \approx 5803\,K$$

Q.6. A solid copper sphere (density r and specific heat c) of radius r at an initial temperature 200 K is suspended inside a chamber whose walls are at almost 0K. The time required for the temperature of the sphere to drop to 100K is ....

Ans. 1.71ρrc

Solution. The energy emitted per second when the temperature of the copper sphere is T and the surrounding temperature T_0

$$= \sigma (T^4 - T_0^4) \times A = \sigma . T^4 A \quad [\because T_0 = 0] \qquad ...(i)$$

We know that

$$dQ = mcdT \Rightarrow \frac{dQ}{dt} = mc\frac{dT}{dt} \qquad ...(ii)$$

From (i) and (ii)

$$\sigma T^4 A = mc\frac{dT}{dt}$$

$$\Rightarrow \quad dt = \frac{mcdT}{\sigma T^4 A} = \frac{\rho \times \frac{4}{3}\pi r^3 cdT}{\sigma T^4 \times 4\pi r^2} \quad \left[\because \ m = \rho \times \frac{4}{3}\pi r^3\right]$$

$$\Rightarrow \quad dt = \frac{\rho rc}{3\sigma}\frac{dT}{T^4}$$

Integrating both sides

$$\int_0^t dt = \frac{\rho rc}{3\sigma}\int_{200}^{100}\frac{dT}{T^4} = \frac{\rho rc}{3\sigma}\left[-\frac{1}{3T^3}\right]_{200}^{100}$$

$$t = -\frac{\rho rc}{9\sigma}\left[\frac{1}{(100)^3} - \frac{1}{(200)^3}\right]$$

$$t = \frac{7\rho rc}{(72\times 10^6)\sigma} \approx \frac{7\rho rc}{72\times 10^6(5.67\times 10^{-8})} = 1.71\rho rc$$

Q.7. A point source of heat of power P is placed at the centre of a spherical shell of mean radius R. The material of the shell has thermal conductivity K. If the temperature difference between the outer and inner surface of the shell in not to exceed T, the thickness of the shell should not be less than ....

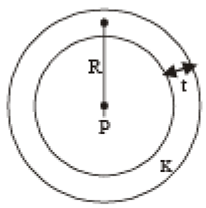
Ans. $t = \frac{4\pi R^2 KT}{P}$

Solution.

KEY CONCEPT :

When the spherical shell is thin, t << R. In this case, The rate of flow of heat from the sphere to the surroundings

$$P = \frac{K(4\pi R^2)T}{t}$$



where T is the temperature difference and t is the thickness of steel then

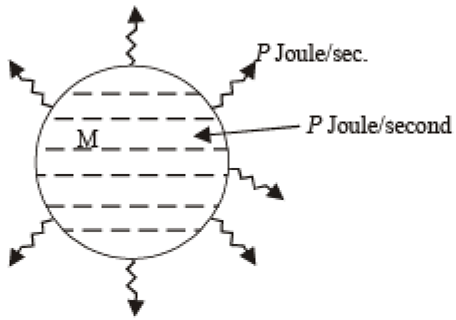
$$t = \frac{4\pi R^2 KT}{P}$$

Q.8. A substance of mass M kg requires a power input of P watts to remain in the molten state at its melting point. When the power source is turned off, the

sample completely solidifies in time t seconds. The latent heat of fusion of the substance is ....

Ans. $L_{fusion} = \frac{P \times t}{m}$

Solution. Since P joules per second of heat is supplied to keep the substance in molten state, it means that the substance in the molten state at its melting point releases P Joule of heat in one second.



The power is turned off then the heat input becomes zero.

But heat output continues. It takes t seconds for the substance to solidify (given). Therefore total heat released in t seconds = $P \times t = mL_{fusion}$

$$L_{fusion} = \frac{P \times t}{m}$$

Q.9. A container of volume 1m^3 is divided into two equal parts by a partition. One part has an ideal gas at 300K and the other part is vacuum. The whole system is thermally isolated from the surroundings. When the partition is removed, the gas expands to occupy the whole volume. Its temperature will now be.......

Ans. 300 K

Solution. In this expansion, no work is done because the gas expands in vacuum. Therefore $\Delta W = 0$

As the process is a adiabatic, Q = 0. From first law of thermodynamics, $\Delta U = 0$ i.e. temperature remains constant.

Q.10. An ideal gas with pressure P, volume V and temperature T is expanded isothermally to a volume 2V and a final pressure P_i. If the same gas is expanded adiabatically to a volume 2V, the final pressure is P_a. The ratio of the specific heats of the gas is 1.67 . The ratio Pa/R is ...

Ans. 0.628

Solution. For isothermal expansion

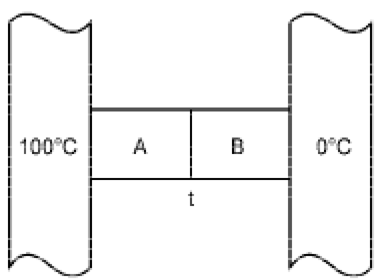
$$P \times V = P_i \times 2V \Rightarrow P_i = \frac{P}{2}$$

For adiabatic expansion

$$PV^{\gamma} = P_a \times (2V)^{\gamma} \Rightarrow P_a = \frac{P}{2^{\gamma}} = \frac{P}{2^{1.67}}$$

$$\therefore \quad \frac{P_a}{P_i} = \frac{P}{2^{1.67}} \times \frac{2}{P} = \frac{2}{2^{1.67}} = 0.628$$

Q.11. Two metal cubes A and B of same size are arranged as shown in Figure. The extreme ends of the combination are maintained at the indicated temperatures. The arrangement is thermally insulated. The coefficients of thermal conductivity of A and B are 300 W/m °C and 200 W/m°C, respectively. After steady state is reached the temperature t of the interface will be .....



Ans. 60°C **Solution.**

Solution. The heat transferred through A per second

$Q_1 = K_1 A (100 - t)$

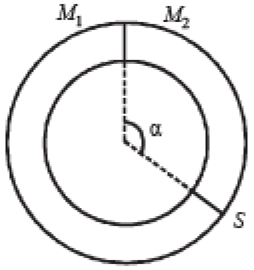
The heat transferred through B per second

$Q_2 = K_2\ A\ (t - 0)$

At steady state $K_1\ A\ (100 - t) = K_2 A\ (t - 0)$

$\Rightarrow 300\ (100 - t) = 200\ (t - 0) \Rightarrow 300 - 3t = 2t \Rightarrow t = 60°\ C$

Q.12. A ring shaped tube contains two ideal gases with equal masses and relative molar masses $M_1 = 32$ and $M_2 = 28$. The gases are separated by one fixed partition and another movable stopper S which can move freely without friction inside the ring. The angle a as shown in the figure is .... degrees.



Ans. 192°

Solution. The movable stopper will adjust to a position with equal pressure on either sides. Applying ideal gas equation to the two gases, we get

$$PV_1 = n_1RT = \frac{m}{M_1}RT,\ PV_2 = n_2RT = \frac{m}{M_2}RT$$

$$\text{Hence, } \frac{V_2}{V_1} = \frac{M_1}{M_2} = \frac{32}{28} = \frac{8}{7}$$

$$\alpha = \frac{360°}{(8+7)} \times 8 = 192°$$

Q.13. Earth receives 1400 W/m ^{2} of solar power. If all the solar energy falling on a lens of area 0.2 m^2 is focused on to a block of ice of mass 280 grams, the time taken to melt the ice will be... minutes. (Latent heat of fusion of ice = 3.3×10^5 J/ kg.

Ans. 5.5 min.

Solution. Solar power received by earth = 1400 W/m^2

Solar power received by 0.2 m^2 area

= (1400 W/m^2) (0.2 m^2) = 280 W

Mass of ice = 280 g = 0.280 kg

Heat required to meltice .

= (0.280) (3.3 × 105) = 9.24 × 104 J

True / False

Q.1. The root-mean square speeds of the molecules of different ideal gases, maintained at the same temperature are the same.

Ans. F

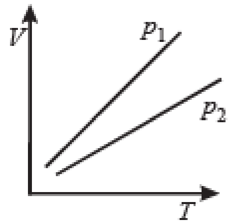
Solution.

KEY CONCEPT : $c = \sqrt{\frac{3RT}{M}}$

At the same temperature $c \propto \frac{1}{\sqrt{M}}$

i.e., dependent on molar mass and hence rms speed c will be different for different ideal gases.

Q.2. The volume V versus temperature T graphs for a certain amount of a perfect gas at two pressure p_1 and p_2 are as shown in Fig. It follows from the graphs that p_1 is greater than p_2.



Ans. F

Solution. For a particular temperature T

$V \propto \frac{1}{P}$

Volume is greater for pressure P_1

$\therefore \quad p_1 < p_2$

Q.3. Two different gases at the same temperature have equal root mean square velocities.

Ans. F

Solution. For a particular termperature $C_{rms} \propto \frac{1}{\sqrt{M}}$

i.e., C_{rms} will have different values for different gases.

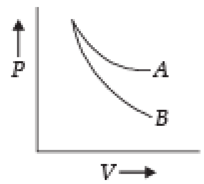
Q.4. The ratio of the velocity of sound in Hydrogen gas $(\gamma = \frac{7}{5})$ to that in Helium gas $(\gamma = \frac{5}{3})$ at the same temperature is $\sqrt{\frac{21}{5}}$.

Ans.F

Solution. $$\frac{(C_{H_2})_1}{(C_{H_e})_2} = \frac{\sqrt{\frac{\gamma_1 RT}{M_1}}}{\sqrt{\frac{\gamma_2 RT}{M_2}}} = \sqrt{\frac{\gamma_1}{\gamma_2} \times \frac{M_2}{M_1}}$$

$$= \sqrt{\frac{7/5}{5/3} \times \frac{4}{2}} = \sqrt{\frac{7}{5} \times \frac{3}{5} \times 2} = \sqrt{\frac{42}{25}}$$

Q.5. The curves A and B in the figure shown P-V graphs for an isothermal and an adiabatic process for an ideal gas. The isothermal process is represented by the curve A.



Ans. T

Solution. The slope of P-V curve is more for adiabatic process than for isothermal process. From the graph it is clear that slope for B is greater than the slope for A.

Q.6. At a given temperature, the specific heat of a gas at constant pressure is always greater than its specific heat at constant volume.

Ans. T

Solution. $C_p - C_v = R$

Q.7. The root mean square (rms) speed of oxygen molecules (O_2) at a certain temperature T (degree absolute) is V. If the temperature is doubled and oxygen gas dissociates into atomic oxygen, the rms speed remains unchanged.

Ans.F

Solution. We know that

$$v = \sqrt{\frac{3RT}{M}} \text{ then } v' = \sqrt{\frac{3R(2T)}{M/2}}$$

$$\therefore \quad v' = 2v$$

Q.8. Two spheres of the same meterial have radii 1 m and 4 m and temperatures 4000K and 2000K respectively. The energy radiated per second by the first sphere is greater than that by the second.

Ans. F

Solution. Energy radiated per second by the first sphere

$$E_1 = \varepsilon\sigma T^4 A = \varepsilon\sigma (4000)^4 \times 4\pi \times 11 \times 1$$
$$= 1024 \times \pi \times 10^{12} \times \varepsilon\sigma$$

Energy radiated per second by the second sphere

$$E_2 = \varepsilon\sigma \times (2000)^4 \times 4\pi \times 4 \times 4$$
$$= 1024\,\pi \times 10^{12} \times \varepsilon\sigma$$
$$E_1 = E_2$$

Subjective Question

Q.1.A sinker of weight w_0 has an apparent weight w, when weighed in a liquid at a temperature t_1 and w_2 when weight in the same liquid at temperature t,. The coefficient of cubical expansion of the material of sinker is β. What is the coefficient of volume expansion of the liquid.

Ans. $\frac{w_2 - w_1}{(w_0 - w_2)(t_2 - t_1)} + \frac{\beta(w_0 - w_1)}{(w_0 - w_2)}$

Solution.

$$W_0 - W_1 = V \times d_\ell \times g \quad \text{...(i)}$$

$$W_0 - W_2 = V' \times d'_\ell \times g \quad \text{...(ii)}$$

Also, $V' = V(1 + \beta\,\Delta T)$ ...(iii)

and $d_\ell = d_\ell'(1 + \gamma_\ell \Delta T)$ ...(iv)

From (ii), (iii) and (iv)

$$W_0 - W_2 = \frac{V(1+\beta\Delta T) \times d_\ell}{1 + \gamma_\ell \Delta T} \times g \quad \text{...(v)}$$

Dividing (i) and (v), we get

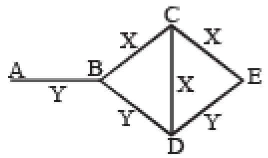
$$\frac{W_0 - W_1}{W_0 - W_2} = \frac{V d_\ell g(1 + \gamma_\ell \Delta T)}{V(1+\beta\Delta T) d_\ell g}$$

$$\Rightarrow \frac{W_0 - W_1}{W_0 - W_2} = \frac{1 + \gamma_\ell \Delta T}{1 + \beta\Delta T} \Rightarrow \frac{W_0 - W_1}{W_0 - W_2} = \frac{1 + \gamma_\ell (t_2 - t_1)}{1 + \beta(t_2 - t_1)}$$

$$\Rightarrow (W_0 - W_1)[1 + \beta(t_2 - t_1) = (W_0 - W_2)[1 + \gamma_\ell (t_2 - t_1)]$$

$$\Rightarrow \gamma_\ell = \frac{W_2 - W_1}{(W_0 - W_2)(t_2 - t_1)} + \frac{\beta(W_0 - W_1)}{(W_0 - W_2)}$$

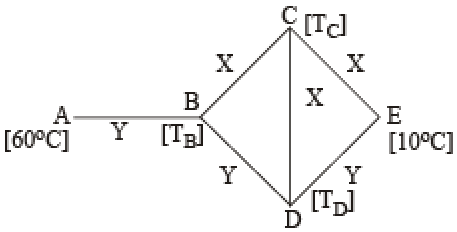
Q.2. Three rods of material X and three rods of material Y are connected as shown in the figure. All the rods are of identical length and cross-sectional area. If the end A is maintained at 60°C and the junction E at 10°C. Calculate the temperature of the junctions B, C and D. The thermal conductivity of X is 0.92 cal/sec-cm-°C and that of Y is 0.46 cal/sec-cm-°C.



Ans. $T_B = 30°C$, $T_C = T_D = 20°C$

Solution. $K_X = 0.92$ cal/sec-cm-°C

$K_Y = 0.46$ cal/sec-cm-°C



NOTE THIS STEP : The heat flow through AB is divided into two path BC and BD. Symmetry shows that no heat will flow through CD. Therefore

$$\frac{K_Y A(60-T_B)}{\ell} = \frac{K_X A(T_B-10)}{2\ell} + \frac{K_Y A(T_B-10)}{2\ell}$$

On solving the above equation, we get $T_B = 30°C$

As C is a point at the middle of BE therefore temperature at C is 20°C.

Similarly temperature at D is also 20°C.

Q.3. Given samples of 1 c.c. of hydrogen and 1c.c. of oxygen, both at N.T.P. which sample has a larger number of molecules?

Ans. Same

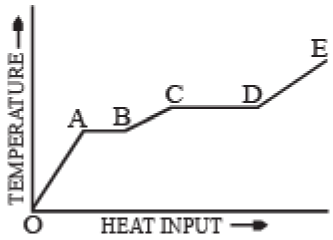
Solution. $PV = nRT$

When P, T are same $n \propto V$

As volumes are same, both samples will have equal number of molecules

Q.4. A Solid material is supplied with heat at a constant rate.

The temperature of the material is changing with the heat input as shown in the graph in figure. Study the graph carefully and answer the following questions :



(i) What do the horizontal regions AB and CD represent?

(ii) If CD is equal to 2AB, what do you infer?

(iii) What does the slope of DE represent?

(iv) The slope of OA > the slope of BC. What does this indicate?

Solution. (i) Region AB : Heat is absorbed by the material at a constant temperature called the melting point.

The phase changes from solid to liquid.

Region CD : Heat is absorbed by the material at a constant temperature called the boiling point. The phase changes from liquid to gas.

(ii) Latent heat of vaporisation = 2 (latent heat of fusion)

(iii) $Q = mc_g \Delta T.$

The slope $DE = \frac{\Delta T}{Q} = \frac{1}{mc_g}$

NOTE : The slope DE indicates that the temperature of the solid begins to rise.

(iv) The reciprocal of heat capacity in solid state is greater than the reciprocal of heat capacity in liquid state

$$\left(\frac{1}{mc}\right)_{solid} > \left(\frac{1}{mc}\right)_{liquid} \Rightarrow (mc)_{liquid} > (mc)_{solid}$$

$$\Rightarrow \; c_{liquid} > c_{solid}$$

Q.5. A jar contains a gas and a few drops of water at T°K. The pressure in the jar is 830 mm of Hg. The temperature of the jar is reduced by 1%. The saturated vapour pressures of water at the two temperatures are 30 and 25 mm of Hg.

Calculate the new pressure in the jar.

Ans. 817 mmHg

Solution.

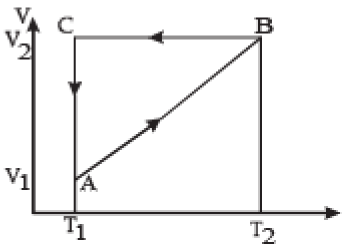
$P_1 = 830 - 30 = 800$ mm Hg ; P_2 ?

$V_1 = V$; $V_2 = V$; $T_1 = T$; $T_2 = T - 0.01\,T = 0.99\,T$

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \quad \therefore P_2 = \frac{P_1T_2}{T_1} = \frac{800 \times 0.099\,T}{T} = 792 \text{ mmHg}$$

$\therefore$ Total pressure in the jar = 792 + 25 = 817 mm Hg

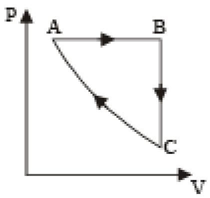
Q.6. A cyclic process ABCA shown in the V-T diagram (fig) is performed with a constant mass of an ideal gas. Show the same process on a P-V diagram



(In the figure, CA is parallel to the V-axis and BC is parallel to the T-axis)

Solution. A → B A straight line between A and B in V∝T graph indicates V ∝ T

⇒ Pressure is constant.



B → C Volume is constant. Since the temperature is decreasing, the pressure should also decrease.

C → A The temperature is constant but volume decreases.

The process is isothermal.

Q.7. A lead bullet just melts when stopped by an obstacle.

Assuming that 25 per cent of the heat is absorbed by the obstacle, find the velocity of the bullet if its initial temperature is 27°C. (Melting point of lead = 327°C, specific heat of lead = 0.03 calories /gm/°C, latent heat of fusion of lead = 6 calories / gm, J = 4.2 joules /calorie).

Ans. 12.96 m/s

Solution. Lead bullet just melts when stopped by an obstacle. Given that 25% of the heat is absorbed by the obstacle. Therefore 75% heat is used in melting of lead. Initial temp. = 27°C M.P. = 300°C

(0.75) K.E. = Heat utilised in increasing the temperature and heat utilised to melt lead at 300°C

$$(0.75)\times\frac{1}{2}Mv^2 = Mc\,\Delta T + ML$$

$$(0.75)\times\frac{1}{2}v^2 = (0.03\times300+6)\times4.2$$

[4.2 to convert into S.I. system] v = 12.96 m/s

Q.8. Calculate the work done when one mole of a perfect gas is compressed adiabatically. The initial pressure and volume of the gas are 105 N/m^2 and 6 litres respectively. The final volume of the gas is 2litre. Molar specific heat of

the gas at constant volume is 3R/2.

Ans. –973.1 J

Solution. Work don in an adiabatic process is

$$W = \frac{1}{1-\gamma}[P_2V_2 - P_1V_1]$$

Here, $P_1 = 10^5 \text{ N/m}^2$, $V_1 = 6\,\ell = 6 \times 10^{-3} \text{ m}^3$

$$P_2 = P_1\left(\frac{V_1}{V_2}\right)^{\gamma}, V_2 = 2\,\ell = 2 \times 10^{-3}$$

Given that $C_v = \frac{3}{2}R$

$\therefore \quad C_p = \frac{5}{2}R \qquad [\because C_p - C_v = R]$

$\therefore \quad \gamma = \frac{C_p}{C_v} = 1.67$

$$\therefore \quad P_2 = 10^5\left[\frac{6}{2}\right]^{1.67} = 10^5 \times (3)^{1.67} = 6.26 \times 10^5 \text{ N/m}^2$$

$$\therefore \quad W = \frac{1}{1-1.67}[6.26 \times 10^5 \times 2 \times 10^{-3} - 10^5 \times 6 \times 10^{-3}]$$

$$W = \frac{1}{-0.67}[1252 - 600] = -\frac{652}{0.67} = -973.1 \text{ J}$$

Work done is negative because the gas is compressed.

Q.9. A solid sphere of copper of radius R and a hollow sphere of the same material of inner radius r and outer radius A are heated to the same temperature and allowed to cool in the same environment. Which of them starts cooling faster ?

Ans. hollow sphere

Solution. NOTE : Since the temperature and surface area is same, therefore the energy emitted per second by both spheres is same.

We know that Q = mcΔT

Since Q is same and c is also same (both copper).

$$\therefore \quad m \propto \frac{1}{\Delta T}$$

Mass of hollow sphere is less;

∴ Temperature change will be more.

∴ Hollow sphere will cool faster.

Q.10. One gram mole of oxygen at 27° and on e atmospheric pressure is enclosed in a vessel.

(i) Assuming the molecules to be moving with Vrms, Find the number of collisions per second which the molecules make with one square metre area of the vessel wall.

(ii) The vessel is next thermally insulated and moved with a constant speed V_o. It is then suddenly stopped. The process results in a rise of the temperature of the gas by 1°C. Calculate the speed V_o.

Ans. 1.97×10^{27}, 35.6 m/s

Solution.

$$F = P \times A = 10^5 \times 1 = 10^5 N$$

$$F = \frac{\Delta p}{\Delta t} \Rightarrow \Delta p = F \times \Delta t = 10^5 \times 1 = 10^5 \quad ...(i)$$

Now, momentum change per second (Δp) = n × 2mv ...(ii)

Where n is the number of collisions per second per square metre area From (i) and (ii)

$$n \times 2mv = 10^5 \qquad \therefore n = \frac{10^5}{2mv}$$

Root mean square velocity

$$v = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3 \times 8.314 \times 300}{32/1000}} = 483.4 \text{ m/s}$$

According to mole concept 6.023×10^{23} molecules will have mass 32 g

$\therefore$ 1 molecule will have mass $\frac{32}{6.023 \times 10^{23}}$ g

$$\therefore \quad n = \frac{10^5 \times 6.023 \times 10^{23}}{2 \times 32 \times 483.4} = 1.97 \times 10^{27}$$

(ii) The kinetic energy of motion of molecules will be converted into heat energy.

K.E. of 1 gm mole of oxygen = $1/2\ Mv^2_0$ ... (i)

where v_0 is the velocity with which the vessel was moving.

The heat gained by 1 gm mole of molecules at constant volume for 1°C rise in temperature = $nC_v\Delta T = 1 \times C_v \times 1 = C_v$ ... (ii)

From (i) and (ii)

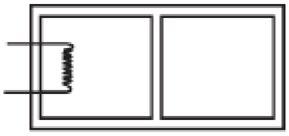
$$\frac{1}{2}Mv_0^2 = C_v \quad \text{But, } C_v = \frac{R}{\gamma - 1}$$

$$\frac{1}{2}Mv_0^2 = \frac{R}{\gamma - 1}$$

$$\therefore \quad v_0 = \sqrt{\frac{2R}{M(\gamma - 1)}} = \sqrt{\frac{2 \times 8.314}{\frac{32}{100} \times (1.41 - 1)}} = 35.6\ \text{ms}^{-1}$$

$[\because \gamma = 1.41 \text{ for } O_2 \text{ (diatomic gas)}]$

Q.11. The rectangular box shown in Fig has a partition which can slide without friction along the length of the box. Initially each of the two chambers of the box has one mole of a mono-atomic ideal gas ($\gamma = 5/3$) at a pressure P_0, volume V_0 and temperature T_0. The chamber on the left is slowly heated by an electric heater. The walls of the box and the partition are thermally insulated. Heat loss through the lead wires of the heater is negligible. The gas in the left chamber expands pushing the partition until the final pressure in both chambers becomes 243 P_0/32. Determine (i) the final temperature of the gas in each chamber and (ii) the work done by the gas in the right chamber.



Ans. 12.9 T_0, 2.25 T_0, –15.58T_0

Solution. For the left chamber

$$\frac{P_0V_0}{T_0} = \frac{P_0 \times 243}{32 \times T_1} \times V_1$$

$$\Rightarrow \quad T_1 = \frac{243}{32} \times \frac{V_1T_0}{V_0} \qquad \text{...(i)}$$

For the right chamber for adiabatic compression

We get, $P_0 V_0^{\gamma} = P_0 \times \frac{243}{32} \times V_2^{\gamma}$

$$\Rightarrow \quad \frac{V_2}{V_0} = \left(\frac{32}{243}\right)^{3/5} \quad \Rightarrow \quad V_2 = \frac{8}{27}V_0$$

But $V_1 + V_2 = 2V_0$

$$\therefore \quad V_1 = 2V_0 - V_2 = 2V_0 - \frac{8}{27}V_0 = \frac{46}{27}V_0 \qquad \text{...(ii)}$$

From (i) and (ii) $T_1 = \frac{243}{32} \times \frac{46 \times V_0}{V_0 \times 27} \times T_0$

or, $T_1 = \frac{207}{16}T_0 = 12.9T_0$ (approx.)

To find the temperature in the second chamber (right), we apply

$$\left(\frac{T_1}{T_2}\right)^{\gamma} = \left(\frac{P_2}{P_1}\right)^{1-\gamma}$$

$$\Rightarrow \quad \left(\frac{T_0}{T_2}\right)^{5/3} = \left(\frac{243\,P_0}{32\,P_0}\right)^{1-5/3} \quad \Rightarrow \quad T_2 = 2.25\,T_0$$

Work done in right chamber (adiabatic process)

$$W = \frac{1}{1-\gamma}(P_2V_2 - P_0V_0)$$

$$= -\frac{3}{2}\left[\frac{243}{32}P_0 \times \frac{8}{27}V_0 - P_0V_0\right]$$

$$= -\frac{3}{2}\left(\frac{9}{4}-1\right)P_0V_0 = -\frac{15}{8} \times RT_0 = -15.8\,T_0$$

Q.12. Two glass bulbs of equal volume are connected by a narrow tube and are filled with a gas at 0°C and a pressure of 76 cm of mercury. One of the bulbs is then placed in melting ice and the other is placed in a water bath maintained at 62°C. What is the new value of the pressure inside the bulbs? The volume of the connecting tube is negligible.

Ans. 83.75 cm Hg

Solution. Let x moles shift from high temperature side to low temperature side.

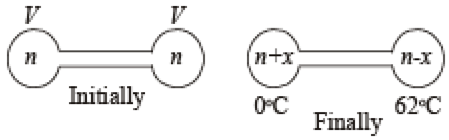
for left bulb PV = nRT

76 × V = nR × 273 Initially

P' × V = (n + x) R × 273 Finally

Dividing, we get

$$\frac{P'}{76} = \frac{n+x}{n} \quad \text{...(i)}$$



For right bulb

76 × V = nR × 273 Initially

P' × V = (n – x) R × 335 Finally .

On dividing,

$$\frac{P'}{76} = \frac{n-x}{x} \times \frac{335}{273} \quad \text{... (ii)}$$

From (i) and (ii)

$$\frac{n+x}{n} = \frac{n-x}{n} \times \frac{335}{273}$$

$$\Rightarrow n = \frac{608}{62} x. \quad \text{... (iii)}$$

Substituting the value of (iii) in (i), we get

$$\frac{P'}{76} = 1 + \frac{62}{608}$$

$$\Rightarrow P' = \frac{670}{608} \times 76 = 83.75 \text{ cm Hg}$$

Q.13. A thin tube of uniform cross-section is sealed at both ends.

It lies horizontally, the middle 5 cm containing mercury and the two equal end containing air at the same pressure P.

When the tube is held at an angle of 60° with the vertical direction, the length of the air column above and below the mercury column are 46cm and 44.5 cm respectively. Calculate the pressure P in centimeters of mercury. (The temperature of the system is kept at 30°C).

Ans. 75.4 cm

Solution.

Let A be the area of cross-section of the tube.

Since temperature is the same, applying Boyle's law on the side AB

$P \times (x \times A) = P_2 \times (x_2 \times A)$ ... (i)

Applying Boyle's law in section CD

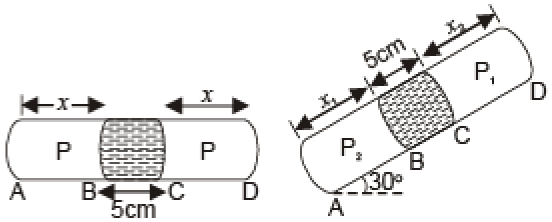
$P \times (x \times A) = P_1 \times (x_1 \times A)$ ... (ii)

From (i) and (ii)

$P_1 \times (x_1 \times A) = P_2 \times (x_2 \times A)$

$\Rightarrow P_1x_1 = P_2x_2$

where P2 = P1 + Pressure due to mercury column



Pressure due to mercury column

$$P = \frac{F}{A} = \frac{mg\sin 30°}{A} = \frac{Vdg\sin 30°}{A}$$

$$= \frac{(A \times 5) \times dg\sin 30°}{A} = 5\sin 30° \text{ cm of Hg}$$

$$P_2 = P_1 + 5\sin 30° = P_1 + 2.5$$

Substituting this value in (iii)

$$P_1 \times x_1 = [P_1 + 2.5] \times x_2$$

$$P_1 \times 46 = [P_1 + 2.5] \times 44.5$$

$$\therefore \quad P_1 = \frac{44.5 \times 2.5}{1.5}$$

Substituting this value in (ii)

$$P \times x = \frac{44.5 \times 2.5}{1.5} \times 46$$

$$\Rightarrow \quad P \times \left[\frac{46 + 44.5}{2}\right] = \frac{44.5 \times 2.5}{1.5} \times 46$$

$$\left[\because x = \frac{x_1 + x_2}{2}\right] \Rightarrow \quad P = 75.4 \text{ cm}$$

Q.14. An ideal gas has a specific heat at constant pressure C_P= 5R/2. The gas is kept in a closed vessel of volume 0.0083 m^3, at a temperature of 300 K and a pressure of 1.6 × 10^6N/ m^2. An amount of 2.49 × 10^4 Joules of heat energy is

supplied to the gas. Calculate the final temperature and pressure of the gas.

Ans. 675 K, 3.6×10^6 N/m^2

Solution. We know that PV = nRT

$$\therefore \quad n = \frac{PV}{RT} = \frac{1.6 \times 10^6 \times 0.0083}{8.3 \times 300} = \frac{16}{3} = 5.33 \text{ moles}$$

$$C_p = \frac{5R}{2} \Rightarrow C_v = \frac{3R}{2}$$

When 2.49×10^4 J of heat energy is supplied at constant volume then we can use the following relationship to find change in temperature.

$$Q = nC_v \Delta T$$

$$\therefore \quad \Delta T = \frac{Q}{nC_v} = \frac{2.49 \times 10^4}{5.33 \times \frac{3}{2} \times 8.3} = 375 \text{ K}$$

Therefore, the final temperature

= 300 + 375 = 675 K

Applying Gay Lussac's Law, to find pressure.

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

$$\Rightarrow \quad P_2 = \frac{P_1 T_2}{T_1} = \frac{1.6 \times 10^6 \times 675}{300} = 3.6 \times 10^6 \text{ Nm}^{-2}$$

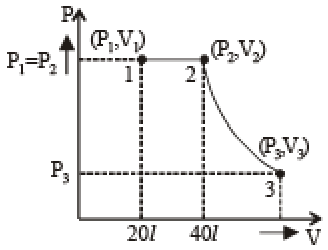
Q.15. Two moles of helium gas (γ = 5/3) are initially at temperature 27°C and occupy a volume of 20 litres. The gas is first expanded at constant pressure until the volume is doubled.

Then it undergoes an adiabatic change until the temperature returns to its initial value.

(i) Sketch the process on a p-V diagram. (ii) What are the final volume and pressure of the gas ? (iii) What is the work done by the gas ?

Ans. (ii) 113 l, 0.44×10^5 N/m^2 (iii) 12450 J

Solution. (i) P – V diagram is drawn below.



(ii) $P_1V_1 = nRT_1$

$\therefore$ $P_1 \times 20 \times 10^{-3} = 2 \times 8.3 \times 300$

$P_1 = 2.49 \times 10^5 \text{ Nm}^{-2}$

Applying $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

For 1 → 2

$$\frac{20}{300} = \frac{40}{T_2} \Rightarrow T_2 = 600 \text{ K}$$

2 → 3 is adiabatic expansion.

$$T_2V_2^{\gamma-1} = T_3V_3^{\gamma-1}$$

$$\therefore \quad V_3 = V_2\left[\frac{T_2}{T_3}\right]^{\frac{1}{\gamma-1}} = 40\left[\frac{600}{300}\right]^{\frac{1}{\frac{5}{3}-1}} = 113\ell$$

$$\left[\because \gamma = \frac{5}{3} \text{ for mono atomic gas}\right]$$

Now, $P_3V_3 = nRT_3$

$$\Rightarrow \quad P_3 = \frac{nRT_3}{V_3} = \frac{2 \times 8.3 \times 300}{113 \times 10^{-3}} = 0.44 \times 10^5 \text{ N/m}^2$$

(NOTE : $T_3 = T_1$ given)

(iii) $W = W_{12} + W_{23}$

$$= P_1(V_2 - V_1) + \frac{nR}{\gamma - 1}(T_2 - T_3)$$

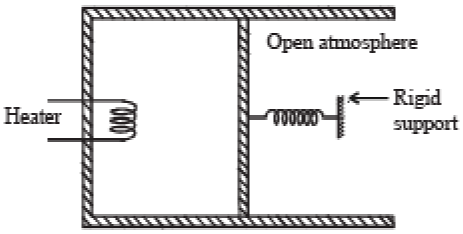
W_{12} = work done at constant pressure

W_{23} = work done in adiabatic condition

$$= 2.49 \times 10^5 (40-20)\,10^{-3} + \frac{2 \times 8.3}{\frac{5}{3}-1}(600-300)$$

= 4980 + 7470 = 12450 J

Q.16. An ideal monatomic gas is confined in a cylinder by a springloaded piston of cross-section 8.0 × 10^{-3} m^2. Initially the gas is at 300 K and occupies a volume of 2.4 × 10^{-3} m^{3}and the spring is in its relaxed (unstretched, uncompressed) state, fig. The gas is heated by a small electric heater until the piston moves out slowly by 0.1 m. Calculate the final temperature of the gas and the heat supplied (in joules) by the heater. The force constant of the spring is 8000 N/m, atmospheric pressure is 1.0 × 10^5 Nm^{-2}. The cylinder and the piston are thermally insulated. The piston is massless and there is no friction between the piston and the cylinder. Neglect heat loss through lead wires of the heater. The heat capacity of the heater coil is negligible. Assume the spring to be massless.

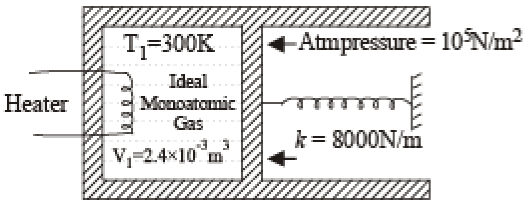


Ans. 800 K, 720 J

Solution. KEY CONCEPT : The final pressure on the gas = atm pressure + pressure due to compression of spring

$$P_2 = P_{atm} + \frac{kx}{A}$$

$$\Rightarrow \quad P_2 = 10^5 + \frac{8000 \times 0.1}{8 \times 10^{-3}} = 2 \times 10^5 \text{ N/m}^2$$



The final volume,

$$V_2 = V_1 + xA$$
$$= 2.4 \times 10^{-3} + 0.1 \times 8 \times 10^{-3} = 3.2 \times 10^{-3}\ m^3$$

Applying $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \Rightarrow T_2 = \frac{P_2V_2T_1}{P_1V_1}$

$$\Rightarrow T_2 = \frac{2 \times 10^5 \times 3.2 \times 10^{-3} \times 300}{10^5 \times 2.4 \times 10^{-3}} = 800\ K.$$

NOTE : Heat supplied by the heater is used for expansion of the gas, increasing its temperature and storing potential energy in the spring.

∴ Heat supplied

$$= P\Delta V + nC_v\Delta T + \frac{1}{2}kx^2$$

$$= 10^5[0.8 \times 10^{-3}] + \frac{P_1V_1}{RT_1}C_v\Delta T + \frac{1}{2}kx^2$$

$$= 80 + \frac{10^5 \times 2.4 \times 10^{-3}}{2 \times 300} \times \frac{3}{2} \times 2 \times 500 + \frac{1}{2} \times 8000 \times 0.1$$
$$= 720\ J$$

Q.17. An ideal gas h aving in itial pressure P, volume V and temperature T is allowed to expand adiabatically until its volume becomes 5.66 V while its temperature falls to T/2 . (i) How many degrees of freedom do the gas molecules have?

(ii) Obtain the work done by the gas during the expansion as a function of the initial pressure P and volume V.

Ans. (i) 5 (ii) 1.25 PV

Solution. (i) Let pressure = P, Volume = V and Temperature = T be the initial quantities and Pressure = P', Volume = 5.66 V Temperature = T/2 be the final quantities.

For adiabatic process

$$TV^{\gamma-1} = \frac{T}{2}(5.66V)^{\gamma-1} \Rightarrow 2 = (5.66)^{\gamma-1}$$

Taking log on both sides, $\log 2 = (\gamma - 1)\log 5.66$

$\Rightarrow \gamma = 1.4$

$$\text{But } \gamma = 1 + \frac{2}{f} \Rightarrow 1.4 = 1 + \frac{2}{f}$$

$$\Rightarrow f = \frac{2}{0.4} = 5$$

Thus degrees of freedom of gas molecules = 5

(ii) For adiabatic process the pressure-volume relationship is

$$P_1V_1^{\gamma} = P_2V_2^{\gamma}$$

$$\Rightarrow P_2 = \frac{P}{(5.66)^{1.4}} = \frac{P}{11.32}$$

Work done for adiabatic process

$$W = \frac{P_1V_1 - P_2V_2}{\gamma - 1} = \frac{PV - \left(\frac{P}{11.32}\right)(5.66V)}{1.4 - 1} = 1.25\,PV$$

Q.18. Three moles of an ideal gas $\left(C_p = \frac{7}{2}R\right)$ at pressure, P_A and temperature T_A is isothermally expanded to twice its initial volume. It is then compressed at constant pressure to its original volume. Finally gas is compressed at constant volume to its original pressure P_A.

(a) Sketch P - V and P - T diagrams for the complete process. (b) Calculate the net work done by the gas, and net heat supplied to the gas during the complete process.

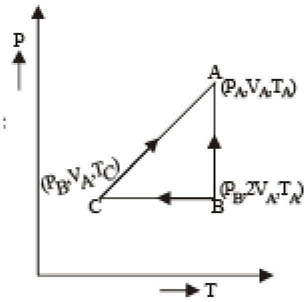
Ans. (b) $0.58\ RT_A$, $0.58\ RT_A$

Solution. (a) Process A to B (isothermal expansion)

$P_AV_A = P_BV_B$

$\Rightarrow \; P_A V_A = P_B \times 2V_A$

$\Rightarrow \; P_B = \dfrac{P_A}{2}$



Process B to C (isobaric compression)

$$\frac{V_B}{T_B} = \frac{V_C}{T_C}$$

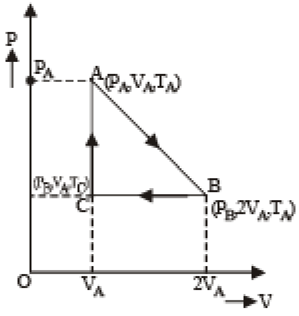
$$\Rightarrow \frac{2V_A}{T_A} = \frac{V_A}{T_C}$$

$$\Rightarrow T_C = \frac{T_A}{2}$$

Process C to A [volume is constant]

$$\frac{P_C}{T_C} = \frac{P_A}{T_A} \Rightarrow \frac{P_B}{T_C} = \frac{P_A}{T_A}$$

$$\Rightarrow \frac{P_A/2}{T_C} = \frac{P_A}{T_A} \quad \Rightarrow T_C = \frac{T_A}{2}$$



Let the system initially be at point A at pressure P_A and temp T_A and volume V_A.

Process A to B

The system is isothermally expanded and reaches a new state B (P_B, $2V_A$, T_A) as shown in the figure.

Process B to C

The system is the compressed at constant pressure to its original volume to reach at state C (P_B, V_A, T_C)

Process C to A Finally at constant volume, the pressure is increased to its original pressure to reach the state A again.

(b) The total work done

$$W = W_{A \to B} + W_{B \to C} + W_{C \to A}$$

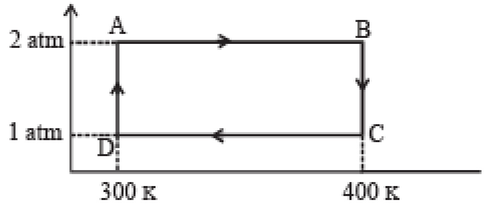
$$= nRT_A \log_e \frac{V_B}{V_A} + nR(T_C - T_A) + 0$$

$$= 2.303 \times 3 \times R \times T_A \log_{10} \frac{2V_A}{V_A} + 3R\left(\frac{T_A}{2} - T_A\right)$$

$$= 2.08\, RT_A - \frac{3}{2} RT_A = 0.58 RT_A$$

NOTE : The total work done is equal to the heat exchanged as the process is cyclic.

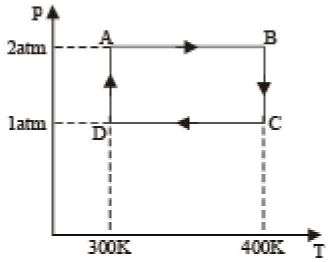
Q.19. Two moles of helium gas undergo a cyclic process as shown in Fig. Assuming the gas to be ideal, calculate the following quantities in this process



(a) The net change in the heat energy (b) The net work done (c) The net change in internal energy

Ans. (a) 1153 J (b) 1153J (c) Zero

Solution. Let us find out the work done in the cycle



Work done from A to B (Isobaric process)

$$W_{AB} = nR(T_B - T_A)$$
$$= nR \times 100 = 2 \times 200 \times 8.32 = 1664\ \text{J}$$

Work done from B to C (Isothermal process)

$$W_{BC} = 2.303nRT \log_{10} \frac{P_B}{P_C}$$

$$= 2.303nR \times 400 \log_{10} \frac{2}{1} = 277.2\, nR$$
$$= 554.4 \times 8.32 = 4612.6$$

Work done from C to D (Isobaric process)

$$W_{CD} = nR(T_D - T_C) = nR(300 - 400)$$
$$= -100nR = -200 \times 8.32 = -1664\ \text{J}$$

Work done from D to A (Isothermal process)

$$W_{DA} = 2.303nRT \log_{10} \frac{P_D}{P_A} = 2.303nR \times 300 \log_{10} \frac{1}{2}$$
$$= -207.9nR$$
$$= -415.8 \times 8.32 = -3459.5\ \text{J}$$

The total work done $= W_{AB} + W_{BC} + W_{CD} + W_{DA}$

$= 1153\ \text{J}$

(a) $\Delta U = Q - W$

For complete cycle $\Delta U = 0$

$\therefore \quad Q = \text{W} = 1153\ \text{J}$

(b) $W = 1153\ \text{J}$

(c) $\Delta U = 0$. Since, the process is cyclic.

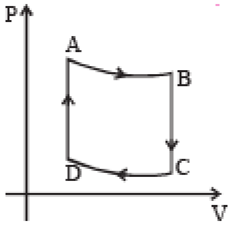
Q.20. One mole of a mono atomic ideal gas is taken through the cycle shown in Fig:

A → B : adiabatic expansion

B → C : cooling at constant volume

C → D : adiabatic compression

D → A : heating at constant volume



The pressure and temperature at A, B, etc. are denoted by P_A, T_A, P_B, T_B etc., respectively. Given that $T_A = 1000$ K, $P_B = (2/3)P_A$ and $P_C = (1/3)P_A$, calculate the following quantities :

(i) The work done by the gas in the process A → B

(ii) The heat lost by the gas in the process B → C.

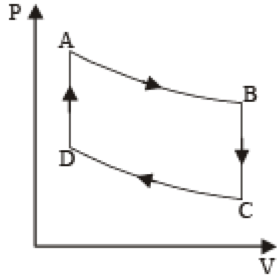
(iii) The temperature T_D. [Given : $(2 / 3)^{2/5} = 0.85$]

Ans. (i) 1870 J (ii) –5298 J (iii) 500 K

Solution. Given $T_A = 1000$ K

$$P_B = \frac{2}{3} P_A$$

$$P_C = \frac{1}{3} P_A$$



(i) W_{AB} (adiabatic expansion)

$$W_{AB} = \frac{nR[T_A - T_B]}{\gamma - 1}$$

Here, $n = 1, R = 8.31\ \text{J mol}^{-1} k^{-1}, T_A = 1000\ \text{K}$

$\gamma = \frac{5}{3}$ (For mono atomic gas)

To find T_B, we use

$$T_A^{\gamma} P_A^{1-\gamma} = T_B^{\gamma} P_B^{1-\gamma} \Rightarrow \left(\frac{P_A}{P_B}\right)^{\gamma-1} = \left(\frac{T_A}{T_B}\right)^{\gamma} \quad \text{... (i)}$$

$$\Rightarrow T_B = T_A \left[\frac{P_A}{P_B}\right]^{\frac{1-\gamma}{\gamma}} = 1000\left[\frac{3}{2}\right]^{\frac{1-5/3}{5/3}} = 850\ \text{K}$$

$$\therefore \quad W_{AB} = \frac{1 \times 8.31[1000 - 850]}{5/3 - 1} = 1870\ \text{J}$$

(ii) Heat Lost B → C

$Q = nC_v \Delta T = nC_v (T_B - T_C)$

Here, $n = 1$, $C_v = \frac{3}{2}R$ (For mono atomic gas),

$T_B = 850\ \text{K}$

To find T_C, we use $\frac{P_B}{T_B} = \frac{P_C}{T_C}$ (volume constant)

$$\Rightarrow \frac{P_C}{P_B} = \frac{T_C}{T_B} \quad \text{..(ii)}$$

$$\Rightarrow T_C = \frac{P_C}{P_B} \times T_B = \frac{1}{2} \times 850 = 425\,K \left[\because \frac{P_C}{P_A} = \frac{\frac{1}{3}P_A}{\frac{2}{3}P_A} = \frac{1}{2}\right]$$

$$\therefore \quad Q = 1 \times \frac{3}{2} \times 8.31\,[425 - 850] = -5298\ \text{J}$$

(iii) Temperature T_D : C to D is adiabatic compression

$$\left(\frac{P_C}{P_D}\right)^{\gamma-1} = \left(\frac{T_C}{T_D}\right)^{\gamma} \quad \text{...(iii)}$$

D to A is isochoric process $\frac{P_D}{T_D} = \frac{P_A}{T_A}$

$$\Rightarrow \frac{P_A}{P_D} = \frac{T_A}{T_D} \quad \text{... (iv)}$$

Multiplying (i) and (iii)

$$\left(\frac{P_C P_A}{P_D P_B}\right)^{\gamma-1} = \left(\frac{T_C}{T_D} \times \frac{T_A}{T_B}\right)^{\gamma} \quad \text{.... (v)}$$

Multiplying (ii) and (iv)

$$\left(\frac{P_A P_C}{P_B P_D}\right) = \left(\frac{T_C T_A}{T_B T_D}\right) \quad \text{... (vi)}$$

From (v) and (vi)

$$\left(\frac{T_C T_A}{T_B T_D}\right)^{\gamma-1} = \left(\frac{T_C T_A}{T_B T_D}\right)^{\gamma} \Rightarrow \frac{T_A T_C}{T_B T_D} = 1$$

$$\Rightarrow T_D = \frac{T_A T_C}{T_B} = \frac{1000 \times 425}{850} = 500K$$

Q.21. An ideal gas is taken through a cyclic thermodynamic process through four steps. The amounts of heat involved in these steps are Q_1 = 5960 J, Q_2 = –5585 J, Q_3 = –2980J and Q_4 = 3645J, respectively. The corresponding quantities of work involved are W_1 = 2200 J, W_2 = – 825 J, W_3 = – 1100 J and W_4 respectively.

1. Find the value of W_4.

2. What is the efficiency of the cycle

Ans. (i) 765 J (ii) 10.82%

Solution. (i) The process is cyclic, therefore $\Delta U = 0$

Now, $\Delta Q = \Delta U + \Delta W$

$\Rightarrow \Delta Q = \Delta W$

$\Rightarrow Q_1 + Q_2 + Q_3 + Q_4 = W_1 + W, + W_3 + W_4$

$\Rightarrow 5960 – 5585 – 2980 + 3645 = 2200 – 825 – 1100 + W_4$

$\Rightarrow W_4 = 765$ J

(ii) Key Concept: $\eta = \dfrac{\text{Work done}}{\text{Heat supplied}}$

$$= \frac{W_1 + W_2 + W_3 + W_4}{Q_1 + Q_4}$$

$$\Rightarrow \quad = \frac{1040}{9605} = 10.82\%$$

Q.22. A closed container of volume 0.02 m ^{3} contains a mixture of neon and argon gases, at a temperature of 27°C and pressure of 1×10^5 Nm^{-2}. The total mass of the mixture is 28 g. If the molar masses of neon and argon are 20 and 40 g mol^{-1} respectively, find the masses of the individual gases in the container assuming them to be ideal (Universal gas constant R = 8.314 J/mol – K).

Ans. Mass of Neon = 4gm, mass of Argon = 24gm

Solution. The total pressure exerted by the mixture P = 10^5 Nm^{-2} Temperature T = 300 K ; Volume = 0.02 m^3 Let there be x gram of Ne. Then mass of Ar will be 28 – x.

$$\text{Number of moles of Neon} = \frac{x}{20};$$

$$\text{Number of moles of Argon} = \frac{28 - x}{40}$$

Partial pressure due to Neon;

$$p_1 = \frac{(x/20)RT}{V}$$

Partial pressure due to Argon

$$p_2 = \frac{[(28-x)/40]RT}{V}$$

But according to Dalton's law of partial pressure

$$P = p_1 + p_2$$
$$10^5 = \frac{xRT}{20V} + \frac{(28-x)RT}{40V}$$

$$\Rightarrow \frac{10^5 \times 40 \times 0.02}{8.314 \times 300} = x + 28 \Rightarrow x = 4g$$

⇒ Mass of Neon = 4g

∴ Mass of Argon = 24g

Q.23. A gaseous mixture enclosed in a vessel of volume V consists of one mole of a gas A with $\gamma (= C_p / C_v)$ = 5 / 3 and another gas B with γ = 7/5 at a certain temperature T. The relative molar masses of the gases A and B are 4 and 32, respectively.The gases A and B do not react with each other and are assumed to be ideal. The gaseous mixture follows the equation $PV^{19/13}$ = constant, in adiabatic processes.

(a) Find the number of moles of the gas B in the gaseous mixture.

(b) Compute the speed of sound in the gaseous mixture at T = 300 K.

(c) If T is raised by 1K from 300 K, find the % change in the speed of sound in the gaseous mixture.

(d) The mixture is compressed adiabatically to 1/5 of its initial volume V. Find the change in its adiabatic compressibility in terms of the given quantities.

Ans. (a) 2 mole (b) 400.03 m/s (c) 1/6 (d) $-8.27´10^{-5}V$

Solution.

(a) $\frac{5+7n_B}{3+5n_B} = \frac{19}{13} \Rightarrow n_B = 2\text{ mol.}$

We know that

$$\frac{n_A+n_B}{\gamma_m - 1} = \frac{n_A}{\gamma_A - 1} + \frac{n_B}{\gamma_B - 1}$$

where γm = Ratio of specific heats of mixture

Here, $n_A = 1,\ \gamma_A = 5/3,\ \gamma_B = 7/5$

According to the relationship

$$PV^{\frac{19}{13}} = \text{constant, we get } \gamma_m = \frac{19}{13}$$

(b) On substituting the values we get nB = 2 mol.

We know that velocity of sound in air is given by the relationship

$$v = \sqrt{\frac{\gamma P}{d}} \text{ where } d = \text{density} = \frac{m}{V}$$

$$\text{Also, } PV = (n_A + n_B)RT \Rightarrow PV = \frac{(n_A+n_B)}{V}RT$$

$$\therefore \quad v = \sqrt{\frac{\gamma(n_A+n_B)RT}{V \times \frac{m}{V}}} = \sqrt{\frac{\gamma(n_A+n_B)RT}{m}}$$

$$\text{Mass of the gas, } m = n_A M_A + n_B M_B = 1 \times 4 + 2 \times 32$$
$$= 68\text{ g/mol} = 0.068\text{ kg/mol}$$

$$\therefore \quad v = \sqrt{\frac{19(1+2) \times 8.314 \times 300}{13 \times 0.068}} = 400.03\text{ ms}^{-1}$$

(c) Velocity of sound,

$$v = \sqrt{\frac{\gamma RT}{M}} \text{ and } v + \Delta v = \sqrt{\frac{\gamma R(T+\Delta T)}{M}}$$

$$\Rightarrow \frac{v+\Delta v}{v} = \sqrt{\frac{T+\Delta T}{T}} = \left(1+\frac{\Delta T}{T}\right)^{1/2}$$

When $\Delta T << T$ then $\frac{\Delta T}{T} << 1$

$$\therefore \quad 1+\frac{\Delta v}{v} = 1+\frac{1}{2}\frac{\Delta T}{T}$$

Percentage change $\frac{\Delta v}{v}\times 100 = \frac{1}{2}\times\frac{\Delta T}{T}\times 100$

$$\frac{\Delta v}{v}\times 100 = \frac{1}{2}\frac{1}{300}\times 100 = \frac{1}{6}\%$$

(d) $PV^{\gamma} = \text{Const.}$

Differentiating the above equation

$$V^{\gamma}(dP) - P(\gamma V^{\gamma-1}\,dV) = 0$$

$$\Rightarrow \quad V^{\gamma}\,dP = \gamma P V^{\gamma-1} dV$$

$$\Rightarrow \quad \frac{dP}{dV} = \frac{\gamma P V^{\gamma-1}}{V^{\gamma}} = \gamma P V^{\gamma-1-\gamma} = \frac{\gamma P}{V}$$

$$\Rightarrow \quad \frac{-dP}{dV/V} = -\gamma\, P$$

$\therefore$ Bulk Modulus $B = \gamma\, P$

$\therefore$ Compressibility $K = \frac{1}{B} = \frac{1}{\gamma P}$

$\therefore$ $K_1 = \frac{1}{\gamma P_1}$ and $K_2 = \frac{1}{\gamma P_2}$

$$\Delta K = K_2 - K_1 = \frac{1}{\gamma P_2} - \frac{1}{\gamma P_1} = \frac{1}{\gamma}\left(\frac{1}{P_2} - \frac{1}{P_1}\right)$$

Since the process is adiabatic, $P_2 V_2^{\gamma} = P_1 V_1^{\gamma}$

$$\therefore \quad P_2 = P_1\left(\frac{V_1}{V_2}\right)^{\gamma} = P_1\left(\frac{V_1}{V_1/5}\right)^{\gamma} = P_1 5^{\gamma}$$

$$\therefore \quad \Delta K = \frac{1}{\gamma}\left(\frac{1}{P_1 5^{\gamma}} - \frac{1}{P_1}\right) = \frac{1}{\gamma P_1}\left(\frac{1}{5^{\gamma}} - 1\right)$$

$$P_1 = \frac{(n_A + n_B)RT}{V} = \frac{(1+2)\times 8.31 \times T}{V} = \frac{24.93T}{V}$$

$$\Rightarrow \Delta K = \frac{1}{\frac{19}{13}\times 24.93 \times \frac{T}{V}}\left(\frac{1}{5^{19/13}} - 1\right)$$

$= -8.27 \times 10^{-5}$ V [∴ T = 300 K]

Q.24. At 27°C two moles of an ideal monoatomic gas occupy a volume V. The gas expands adiabaticallly to a volume 2V. Calculate (i) the final temperature of the gas, (ii) change in its internal energy, and (iii) the work done by the gas during this process.

Ans. (i) 189 K (ii) –2767 J (iii) 2767 J

Solution.

(i) $T_1 = 27 + 273 = 300\,K;\ \gamma = \frac{5}{3}$ (for monoatomic gas)

$V_1 = V$

$V_2 = 2V$

$T_2 = ?$

Since the gas expands adiabatically.

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$$

$$\Rightarrow T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = 300\left[\frac{V}{2V}\right]^{5/3-1} = 189\,K$$

(ii) $$W = \frac{-nR(T_2 - T_1)}{\gamma - 1} = \frac{-2\times 8.31(189-300)}{5/3-1}$$

$$= \frac{+8.31\times 111\times 3}{2} = +2767J$$

Change in internal Energy According to first law of thermodynamics

$\Delta Q = \Delta U + \Delta W$ But $\Delta Q = 0$ (as the process is adiabatic)

$\therefore \Delta U = -\Delta W = -2767$ J

(iii) W = 2767 J

Q.25. The temperature of 100g of water is to be raised from 24°C to 90°C by adding steam to it. Calculate the mass of the steam required for this purpose.

Ans. 0.0122 Kg

Solution. Heat lost by steam = Heat gained by water

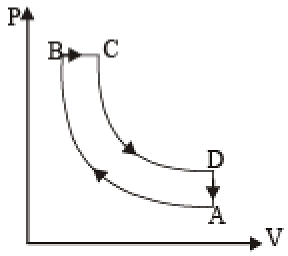
$$m_s L_{fus} = m_w c\, \Delta T$$

$$\Rightarrow \quad m_s = \frac{m_w c \Delta T}{L_{fus}} = \frac{0.1 \times 4200 \times 66}{540 \times 10^3 \times 4.2} = 0.0122 \text{ kg}$$

Q.26. One mole of a diatomic ideal gas (g = 1.4) is taken through a cyclic process starting from point A. The process A ⇒ B is an adiabatic compression, B ⇒ C is isobaric expansion, C ⇒ D is an adiabatic expansion, and D ⇒ A is isochoric. The volume ratios are V_A / V_B = 16 and V_C / V_B = 2 and the temperature at A is T_A = 300 K.Calculate the temperature of the gas at the points B and D and find the efficiency of the cycle.

Ans. T_B = 909 K, T_D = 791 K, 61.4%

Solution. n = 1, for diatomic gas,



$$\gamma = 1 + \frac{2}{5} = \frac{7}{5} = 1.4$$

$A \rightarrow B$, adiabatic compression
$B \rightarrow C$, isobaric expansion
$C \rightarrow D$, adiabatic expansion
$D \rightarrow A$, isochoric process

Given $\frac{V_A}{V_B} = 16, \frac{V_C}{V_B} = 2$

$T_A = 300\, K, T_B = ?, T_D = ?, \eta = ?$

For adiabatic compression process A ⇒ B

$$T_A V_A^{\gamma-1} = T_B V_B^{\gamma-1} \text{ or}$$

$$T_B = \left(\frac{V_A}{V_B}\right)^{\gamma-1} T_A = (16)^{2/5} \times 300 = 909\,K$$

$\therefore$ For isobaric process $B \to C$: According to Charles' law

$$\text{As } \frac{V_B}{T_B} = \frac{V_C}{T_C} \text{ or } T_C = T_B\left(\frac{V_C}{V_B}\right) = 909\,[2] = 1818\,K$$

For adiabatic expansion process C ⇒ D :

$$\text{As } \frac{V_A}{V_B} = 16 \text{ and } \frac{V_C}{V_B} = 2; \text{ hence } \frac{V_A}{V_C} = 8$$

According to Poisson's law,

$$T_C V_C^{\gamma-1} = T_D V_D^{\gamma-1}$$

$$\therefore \quad T_D = T_C\left[\frac{V_C}{V_D}\right]^{\gamma-1} = 1818\left[\frac{1}{8}\right]^{2/5} = \frac{1818}{(64)^{1/5}} = 791K$$

For $B \to C$ process : Heat absorbed

$$Q_1 = nC_p(T_C - T_B)$$

$$= n\frac{\gamma R}{\gamma-1}(T_C - T_B) = 1\frac{(7/5)R}{(2/5)}(1818-909)$$

$$= \frac{7R}{2} \times 909 \cong 3182\,R$$

For $D \to A$ process : Heat released

$$Q_2 = nC_v(T_D - T_A) = n\frac{R}{\gamma-1}(T_D - T_A)$$

$$= 1.\frac{R}{(2/5)}(791-300) = \frac{5R}{2} \times 491$$

($\therefore$ No heat is exchanged in adiabatic processes).

Now, $W_{AB} = -\frac{nR}{\gamma - 1}(T_B - T_A)$

$$= -\frac{R}{(2/5)}(900-300) = -\frac{5R}{2} \times 609$$

$$W_{BC} = -nR(T_C - T_B) = 1 \times R(1818-909) = 909R$$

$$W_{CD} = -\frac{nR}{\gamma - 1}(T_C - T_D) = +\frac{R}{(2/5)}(1818-791)$$

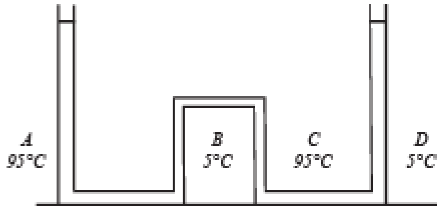
$$= \frac{5R}{2} \times 1027$$

$$W_{net} = 909R + \frac{5R}{2}(1027-609) = 909R + \frac{5R}{2} \times 418$$

$$= 909R + 1045R = 1954R$$

$\therefore$ Efficiency $= 100 \times (W_{net}/Q_1) = 100 \times \frac{1954R}{3182R} = 61.4\%$

Q.27. The apparatus shown in the figure consists of four glass columns connected by horizontal sections. The height of two central columns B and C are 49 cm each. The two outer columns A and D are open to the atmosphere. A and C are maintained at a temperature of 95° C while the columns B and D are maintained at 5°C. The height of the liquid in A and D measured from the base the are 52.8 cm and 51cm respectively. Determine the coefficient of thermal expansion of the liquid.



Ans. 6.67×10^{-5} per °C

Solution. Let the pressure at point O be P_0. Since the liquid is at equilibrium at M

$$P_A + h_1 \rho_{95°} g = P_0 + h\,\rho_{5°}\,g$$

$$\Rightarrow P_0 = P_A + h_1 \rho_{95°}\,g - h\,\rho_{5°}\,g \qquad \text{...(i)}$$

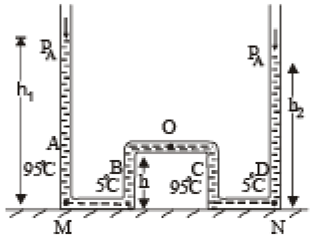
Since the liquid is at equilibrium at N

$$\Rightarrow \quad P_A + h_2 \rho_{5^\circ} g = P_0 + h\, \rho_{95^\circ} g$$
$$\Rightarrow \quad P_0 = P_A + h_2 \rho_{5^\circ} g - h\, \rho_{95^\circ} g \qquad \text{...(ii)}$$

From (i) and (ii)

$$P_A + h_1 \rho_{95^\circ} g - h\, \rho_{5^\circ} g$$
$$= P_A + h_2 \rho_{5^\circ} g - h\, \rho_{95^\circ} g$$

$$\Rightarrow \quad \frac{\rho_{5^\circ}}{\rho_{95^\circ}} = 1.018 \text{ ... (i)}$$



We know that

$$\rho_0 = \rho_1 (1 + \gamma\, \Delta T)$$

Applying the above formula, we get

$$\rho_0 = \rho_{95^\circ} (1 + \gamma \times 95)$$
$$\rho_0 = \rho_{5^\circ} (1 + \gamma \times 5)$$
$$\therefore \quad \frac{\rho_{5^\circ}}{\rho_{95^\circ}} = \frac{1 + 95\gamma}{1 + 5\gamma} \qquad \text{...(ii)}$$

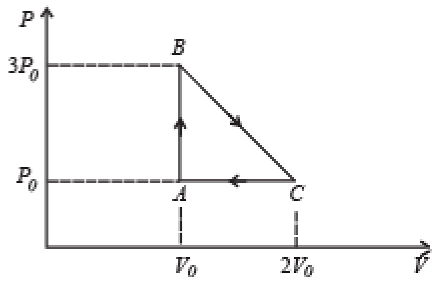
From (i) and (ii)

$$\frac{1 + 95\gamma}{1 + 5\gamma} = 1.018 \quad \Rightarrow \quad \gamma = 2.002 \times 10^{-4}$$

But $\gamma = 3\alpha$

$$\Rightarrow \quad \alpha = \frac{\gamma}{3} = \frac{2.002 \times 10^{-4}}{3} = 6.67 \times 10^{-5}\ {}^\circ C^{-1}$$

Q.28. One mole of an ideal monatomic gas is taken round the cyclic process ABCA as shown in Figure. Calculate



(a) the work done by the gas. (b) the heat rejected by the gas in the path CA and the heat absorbed by the gas in the path AB; (c) the net heat absorbed by the gas in the path BC; (d) the maximum temperature attained by the gas during the cycle.

Ans. (a) P_0V_0 (b) $-\frac{5}{2}P_0V_0$, $3P_0V_0$ (c) $\frac{P_0V_0}{2}$ (d) $\frac{25P_0V_0}{8R}$

Solution. n = 1, For monoatomic gas :

$$C_p = \frac{5R}{2}, C_v = \frac{3R}{2}$$

Cyclic process

$A \to B \Rightarrow$ Isochoric process
$C \to A \Rightarrow$ Isobaric compression

(a) Work done = Area of closed curve ABCA during cyclic process. i.e. ΔABC

$$\Delta W = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} V_0 \times 2P_0 = P_0V_0$$

(b) Heat rejected by the gas in the path CA during isobaric compression process

$$\Delta Q_{CA} = nC_p\Delta T = 1 \times (5R/2)(T_A - T_C)$$

$$T_C = \frac{2P_0V_0}{I \times R}, T_A = \frac{P_0V_0}{I \times R}$$

$$\Delta Q_{CA} = \frac{5R}{2}\left[\frac{P_0V_0}{R} - \frac{2P_0V_0}{R}\right] = -\frac{5}{2}P_0V_0$$

Heat absorbed by the gas on the path AB during isochoric process

$$\Delta Q_{AB} = nC_V\Delta T = 1 \times (3R/2)(T_B - T_A)$$

$$= \frac{3R}{2}\left[\frac{3P_0V_0}{1\times R} - \frac{P_0V_0}{1\times R}\right] = 3P_0V_0$$

(c) As $\Delta U = 0$ in cyclic process, hence,

$\Delta Q = \Delta W$

$\Delta Q_{AB} + \Delta Q_{CA} + \Delta Q_{BC} = \Delta W$

$$\Delta Q_{BC} = P_0V_0 - \frac{P_0V_0}{2} = \frac{P_0V_0}{2}$$

NOTE : As net heat is absorbed by the gas during path BC, temp. will reach maximum between B and C.

(d) Equation for Line BC is $P = -\left[\frac{2P_0}{V_0}\right]V + 5P_0$,

$P = \frac{RT}{V}$ [For one mole]

$$\therefore \quad RT = -\frac{2P_0}{V_0}V^2 + 5P_0V \qquad \text{...(i)}$$

For maximum; $\frac{dT}{dV} = 0$, $-\frac{2P_0}{V_0} \times 2V + 5P_0 = 0$;

$$\therefore \quad V = \frac{5V_0}{4} \qquad \text{...(ii)}$$

Hence from equation (i) and (ii)

$$RT_{max} = \frac{-2P_0}{V_0} \times \left(\frac{5V_0}{4}\right)^2 + 5P_0\left(\frac{5V_0}{4}\right)$$

$$= -2P_0V_0 \times \frac{25}{16} + \frac{25P_0V_0}{4} = \frac{25}{8}P_0V_0$$

$$\therefore \quad T_{max} = \frac{25}{8}\frac{P_0V_0}{R}$$

Q.29. A solid body X of heat capacity C is kept in an atmosphere whose temperature is T_A = 300 K. At time t = 0 the temperature of X is T_0 = 400 K. It cools according to Newton's law of cooling. At time t_1, its temperature is found to be 350 K.

At this time (t,), the body X is connected to a large box Y at atmospheric temperature TA, through a conducting rod of length L, cross-sectional area A and thermal conductivity K. The heat capacity of Y is so large that any variation in its temperature may be neglected. The cross-sectional area A of the connecting rod is small compared to the surface area of X. Find the temperature of X at time t = 3t₁.

Ans. $\left[300+12.5e^{\frac{-2KAt_1}{CL}}\right]$ Kelvin

Solution. Case (i) According to Newton's law of cooling

$$\frac{dT}{dt}=-K'(T-T_A) \Rightarrow \frac{dT}{T-T_A}=-K'dt$$

On integrating, we get

$$\int_{400}^{350}\frac{dT}{T-T_A}=K\int_0^{t_1}dt$$

$$-[\log_e(T-T_A)]_{400}^{350}=K'[t]_0^{t_1}$$

$$\Rightarrow -\log_e\frac{350-300}{400-300}=K't_1$$

$$\Rightarrow \log_e\frac{100}{50}=K't_1 \text{ or } K't_1=\log_e 2 \quad ...(i)$$

Case (ii)

NOTE : When the body X is connected to a large box Y. In this case cooling occurs by Newton's law of cooling as well as by conduction

$$\therefore \quad -\frac{dT}{dt}=K'(T-T_A)+\frac{KA(T-T_A)}{CL}$$

$$\Rightarrow -\frac{dT}{dt}=\left[K'+\frac{KA}{CL}\right](T-T_A) \qquad (\text{for } t>t_1]$$

Where K = coefficient of thermal conductivity of the rod.

$$\Rightarrow \quad \frac{-dT}{T-T_A} = \left[K' + \frac{KA}{CL}\right] dt$$

On integrating, we get

$$-\int_{350}^{T} \frac{dT}{T-T_A} = \int_{t_1}^{3t_1} \left(K' + \frac{KA}{CL}\right) dt$$

$$\Rightarrow \quad -\left[\log_e (T-T_A)\right]_{350}^{T} = \left(K' + \frac{KA}{CL}\right)[t]_{t_1}^{3t_1}$$

$$\Rightarrow \quad \log_e \frac{350-300}{T-300} = \left(K' + \frac{KA}{CL}\right) 2t = 2K' t_1$$

$$\Rightarrow \quad \log_e \frac{50}{T-300} = 2(\log_e 2) + \frac{2KA}{CL} t_1$$

$$\frac{50}{T-300} = e^{\{\log_e 4 + \frac{2KA}{CL} t_1\}}$$

$$\Rightarrow \quad T - 300 = 50\, e^{-[\log_e 4]} \times e^{\frac{-2KAt_1}{CL}}$$

$$\Rightarrow \quad T = [300 + 12.5\, e^{\frac{-2KAt_1}{CL}}] \text{ Kelvin}$$

Q.30. Two moles of an ideal monatomic gas, initially at pressure p, and volume V_1, undergo an adiabatic compression until its volume is V_2. Then the gas is given heat Q at constant volume V_2.

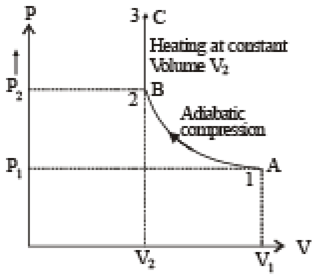
(a) Sketch the complete process on a p – V diagram.

(b) Find the total work done by the gas, the total change in its internal energy and the final temperature of the gas. [Give your answer in terms of p_1, V_1, V_2, Q and R]

Ans. (b) $\frac{3}{2} P_1 V_1 \left[1 - \left(\frac{V_1}{V_2}\right)^{2/3}\right]$, $Q - \frac{3}{2} P_1 V_1 \left[1 - \left(\frac{V_1}{V_2}\right)^{2/3}\right]$, $\frac{P_1 V_1^{5/3} V_2^{-2/3}}{2R} + \frac{Q}{3R}$

Solution. n = no. of moles = 2,

(A) The complete process is shown on P-V diagram in the figure.



(B) (i) Total work done

$$W = W_{AB} + W_{BC} = \frac{(P_1V_1 - P_2V_2)}{(\gamma - 1)} + 0$$

$$[\because W_{BC} = P\Delta V = P \times 0 = 0]$$

According to Poisson's law, $P_2 = P_1\left(\frac{V_1}{V_2}\right)^{\gamma}$

$$\therefore \quad W = \frac{1}{\gamma - 1}\left[P_1V_1 - P_1\left(\frac{V_1}{V_2}\right)^{\gamma} V_2\right]$$

$$= \frac{1}{\gamma - 1}\left[P_1V_1 - P_1V_2 . \frac{V_1}{V_2} . \left(\frac{V_1}{V_2}\right)^{\gamma - 1}\right]$$

For monoatomic gas,

$$\gamma = 1 + \frac{2}{3} = \frac{5}{3},$$

$$W = \frac{3}{2}\left[P_1V_1 - P_1V_1\left(\frac{V_1}{V_2}\right)^{2/3}\right] = \frac{3}{2}P_1V_1\left[1 - \left(\frac{V_1}{V_2}\right)^{2/3}\right]$$

$$\Delta U = \Delta U_{AB} + \Delta U_{BC} = Q - W$$

$$= Q - \frac{3}{2}P_1V_1\left[1 - \left(\frac{V_1}{V_2}\right)^{2/3}\right]$$

[according to first law of thermodynamics]

$$\begin{bmatrix} B \to C & Q = \Delta U_{BC} + 0 \\ A \to B & Q = \Delta U_{AB} + W \end{bmatrix}$$

(iii) For process $BC : \Delta U_{BC} = nC_v \Delta T = Q$

$[\because W_{BC} = 0]$

For monoatomic gas $C_v = \frac{R}{\gamma - 1} = \frac{3}{2}R,$

$$\therefore \quad \Delta U_{BC} = Q = 2 \times \frac{3R}{2}.\Delta T$$

According to Poission's Law :

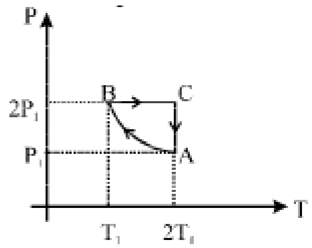
For the process $AB,\ T_A V_B^{\gamma-1} = T_B V_B^{\gamma-1}$

$$\text{or} \quad T_B = T_A\left(\frac{V_1}{V_2}\right)^{\gamma-1} = \frac{P_1 V_1}{nR}\left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

$$\therefore \quad T_B = \frac{P_1}{2R}.V_1^{\gamma}.V_2^{1-\gamma} = \frac{P_1 V_1^{5/3} V_2^{-2/3}}{2R}$$

$$\text{Hence, } T_C = T_B + \Delta T = \frac{P_1 V_1^{5/3} V_2^{-2/3}}{2R} + \frac{Q}{3R}$$

Q.31.Two moles of an ideal monatomic gas is taken through a cycle ABCA as shown in the P-T diagram. During the process AB, pressure and temperature of the gas vary such that PT = Constant. If T_1 = 300 K, calculate



(a) the work done on the gas in the process AB and (b) the heat absorbed or released by the gas in each of the processes.

Give answer in terms of the gas constant R.

Ans. (a) 1200R (b) –2100R, 831.6 R

Solution. For PV^x = Constt., Molar heat capacity

$$C = \frac{R}{\gamma - 1} + \frac{R}{1-x} = \frac{R}{\frac{5}{3}-1} + \frac{R}{1-\frac{1}{2}}$$

Here P^2V = constant or $PV^{1/2}$ = constant

$$\therefore x = \frac{1}{2}$$

$$\Rightarrow C = 3.5R$$

$$Q_{A\to B} = nC\,\Delta T = 2(3.5\,R)(300-600) = -2100\,R$$

Process B – C: Process is isobaric therefore

$$Q_{B\to C} = nC_p\,\Delta T = (2)\left(\frac{5}{2}R\right)(T_C - T_B)$$

$$= 2\left(\frac{5}{2}R\right)(2T_1 - T_1) = (5R)(600-300) = 1500R$$

Heat is absorbed

Process C – A: Process is isothermal

$$\Delta T = 0 \text{ and } Q_{C\to A} = W_{C\to A} = nRT_C \ln\left(\frac{P_C}{P_A}\right)$$

$$= nR(2T_1)\ln\left(\frac{2P_1}{P_1}\right) = (2)(R)(600)\ln(2) = 1200R\times 0.6932$$

$$Q_{C\to A} = 831.6\,R \text{ (absorbed)}$$

Q.32. An ice cube of mass 0.1 kg at 0°C is placed in an isolated container which is at 227°C. The specific heat S of the container varies with temperature T according to the empirical relation S = A + BT, where A = 100 cal/kg-K and B = 2 ´ 10^{-2} cal/kg-K^2. If the final temperature of the container is 27oC, determine the mass of the container. (Latent heat of fusion of water = 8 *10^4 cal/kg, Specific heat of water = 10^3 cal/kg-K).

Ans. 0.495 Kg

Solution.

Here the equilibrium temperature is 273 + 27 = 300 K Also according to the principle of calorimetry Heat lost by container = Heat gained by ice.

Heat lost by container : NOTE : Since specific heat is variable, we need to take the help of calculus to find the heat lost by the container.

Let dQ be the heat lost when the temperature decreases by dT at any instant when the temperature of the container is T.

∴ dQ = mc dT

where m is the mass of the container and C = A + BT is specific heat at that temperature

∴ dQ = m (A + BT) dT

On integrating, we get

$$Q = \int_{500}^{300} m(A + BT)\,dT = m\left[AT + \frac{BT^2}{2}\right]_{500}^{300}$$

= – 21600 m calorie (heat lost)

Heat gained by ice This heat is to be divided into two parts

(i) 0° ice → 0° water

(ii) 0° water → 27° water

$$Q_1 = mL = 0.1 \times 80{,}000 = 8000 \text{ cal} \qquad Q_2 = mc\Delta T = 0.1 \times 10^3 \times 27 = 2700 \text{ cal}$$

$$\therefore \quad Q_1 + Q_2 = 8000 + 2700 = 10{,}700 \text{ cal} \qquad \text{... (i)}$$

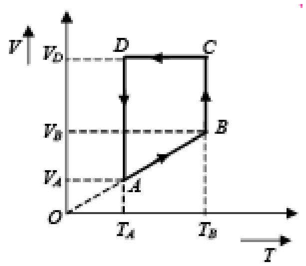
Heat lost = heat gained

21600 m = 10,700

⇒ m = 0.495 kg

Q.33. A monoatomic ideal gas of two moles is taken through a cyclic process starting from A as shown in figure. The volume

ratios are $\frac{V_B}{V_A} = 2$ and $\frac{V_D}{V_A} = 4$. If the temperature T_A at A is **27°C.**



Calculate, (a) the temperature of the gas at point B, (b) heat absorbed or released by the gas in each process, (c) the total work done by the gas during the complete cycle.

Express your answer in terms of the gas constant R.

Ans. (a) 600 K (b) 1500R, 831.8R, –900R, –831.8R (c) 600R

Solution.

(a) Since AB is a straight line in V-T graph

$$\therefore \quad \frac{V}{T} = \text{Constant (Isobaric process)}$$

$$\therefore \quad \frac{V_A}{T_A} = \frac{V_B}{T_B}$$

$$T_B = \frac{V_B}{V_A} \times T_A = 2 \times 300 = 600\,K \qquad \left[\because \frac{V_B}{V_A} = 2\right]$$

(b) (i) A to B is a isobaric process

$$\therefore \quad Q = nC_p\Delta T = 2 \times \frac{5}{2}R \times 300$$

$$= 1500\,R \qquad \left[\because C_p = \frac{5}{2}R \text{ for monoatomic gas}\right]$$

NOTE : Heat is absorbed as Q is positive. (ii) B to C is an isothermal process.

Since the temperature is not changing

∴ Internal energy change dU = 0

∴ From first law of thermodynamics dQ = dW

$$\therefore \quad Q = 2.303 \times nRT \log_{10} \frac{V_f}{V_i}$$
$$= 2.30. \times 2 \times R \times 600 \times \log_{10} 2$$
$$= 2763.6 \times \log_{10} 2 \times R = 831.8\,R$$

NOTE : Heat is absorbed since temperature is same but volume increases.

(iii) C To D is a isochoric process

$$\therefore \quad dW = 0$$
$$\therefore Q = nC_v \Delta T = n\left(\frac{3}{2}R\right)(T_A - T_B)$$
$$= 2 \times \frac{3}{2} R \times (-300) = -900\,R$$

Volume is constant as heat is released.

(iv) D to A is an isothermal process

$$\therefore Q = 2.303 \times nRT \log_{10} \frac{V_f}{V_i}$$
$$= 2.303 \times 2\,R \times 300 \times \log\left(\frac{V_f}{V_i}\right) = -831.8\,R$$

Heat is released as Q is positive.

(c) Total work done

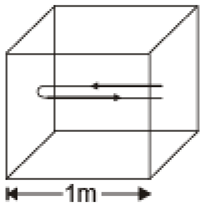
$$= Q_{A \to B} + Q_{B \to C} + Q_{C \to D} + Q_{D \to A}$$
$$= (1500\,R + 831.8\,R) - (900\,R + 831.8\,R) = 600\,R$$

Q.34. A cubical box of side 1 meter contains helium gas (atomic weight 4) at a pressure of 100 N/m2. During an observation time of 1 second, an atom travelling with the root-meansquare speed parallel to one of the edges of the cube, was found to make 500 hits with a particular wall, without any collision with other atoms. Take R = 25/3 J/mol-K and k = 1.38 × 10^{-23} J/K

(a) Evaluate the temperature of the gas. (b) Evaluate the average kinetic energy per atom. (c) Evaluate the total mass of helium gas in the box.

Ans. (a) 160K (b) 3.312 × 10–21 J (c) 0.3012 gm

Solution. The distance travelled by an atom of helium in 1/500sec (time between two successive collision) is 2m. Therefore root mean square speed



$$C_{rms} = \frac{\text{distance}}{\text{time}} = \frac{2}{1/500} = 1000 \text{ m/s}$$

(a) But $C_{rms} = \sqrt{\frac{3RT}{M}}$

$$\Rightarrow \quad 1000 = \sqrt{\frac{3 \times 25/3 \times T}{4 \times 10^{-3}}} \Rightarrow T = 160\text{K}$$

(b) Average kinetic energy of an atom of a monoatomic

gas $= \frac{3}{2}kT$

$$\therefore E_{av} = \frac{3}{2}kT = \frac{3}{2} \times \left(1.38 \times 10^{-23}\right) \times 160$$
$$= 3.312 \times 10^{-21} \text{ Joules}$$

(c) From gas equation $PV = \frac{m}{M}RT$

$$m = \frac{PVM}{RT} = \frac{100 \times 1 \times 4}{25/3 \times 160} \Rightarrow m = 0.3012 \text{ gm}$$

Q.35. An insulated container containing monoatomic gas of molar mass m is moving with a velocity v_0. If the container is suddenly stopped, find the change in temperature.

Ans. $mv^2_0/3R$

Solution. When container is stopped, velocity decreases by v_0.

Therefore change in kinetic energy = $1/2(nm)v^2_0$ ... (i)

Here n = number of moles of gas present in the container.

The kinetic energy at a given temperature for a monatomic

gas is = 3/2 *nRT

∴ Change in kinetic energy = $\frac{3}{2} \times nR(\Delta T)$ .... (ii)

where ΔT = Change in temperature From (i) and (ii)

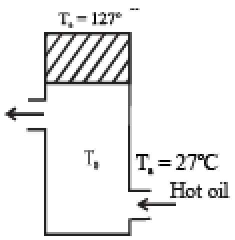
$$\frac{3}{2}nR(\Delta T) = \frac{1}{2}(nm)v_0^2 \quad \therefore \quad \Delta T = \frac{mv_0^2}{3R}$$

Q.36. Hot oil is circulated through an insulated container with a wooden lid at the top whose conductivity

K = 0.149 J/(m-°C-sec), thickness t = 5 mm, emissivity = 0.6.

Temperature of the top of the lid is maintained at T_l = 127°C.

If the ambient temperature T_a = 27°C.



Calculate : (a) rate of heat loss per unit area due to radiation from the lid. (b) temperature of the oil.

(Given $\sigma = \frac{17}{3} \times 10^{-8}\ Wm^{-2}K^{-4}$)

Ans. (a) 595 W/m^2 (b) 419.83 K

Solution. (a) The rate of heat loss per unit area per second due to radiation is given by Stefan's-Boltzmann law

$$E = e\sigma(T^4 - T_0^4)$$

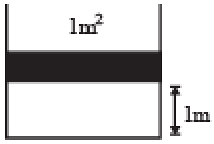
$$= 0.6 \times \frac{17}{3} \times 10^{-8}[(400)^4 - (300)^4] = 595 \text{ watt/m}^2$$

(b) Let T_{oil} be the temperature of the oil.

Then rate of heat flow through conduction = Rate of heat flow through radiation

$$\Rightarrow T_{oil} = \frac{595 \times \ell}{k} + T = \frac{595 \times 5 \times 10^{-3}}{0.149} + 400 = 419.83 \text{ K}$$

Q.37. A diatomic gas is enclosed in a vessel fitted with massless movable piston. Area of cross section of vessel is 1 m^2.Initial height of the piston is 1 m (see the figure). The initial temperature of the gas is 300 K. The temperature of the gas is increased to 400 K, keeping pressure constant, calculate the new height of the piston. The piston is brought to its initial position with no heat exchange. Calculate the final temperature of the gas. You can leave answer in fraction.



Ans. $\frac{4}{3}$m, $400\left(\frac{4}{3}\right)^{2/5}$ K

Solution. At constant pressure, we have $\frac{T_1}{V_1} = \frac{T_2}{V_2}$

also, V = A × h

$$\therefore \frac{T_1}{Ah_1} = \frac{T_2}{Ah_2}$$

$$\Rightarrow h_2 = \frac{T_2 h_1}{T_1} = \frac{400}{300} \times 1 = \frac{4}{3} \text{m}$$

when the gas is compressed without heat exchange, the process is adiabatic

$$\therefore T'_1 = T_2\left(\frac{V_2}{V_1}\right)^{\gamma-1} = 400\left(\frac{4}{3}\right)^{2/5} K$$

Q.38. A small spherical body of radius r is falling under gravity in a viscous medium. Due to friction the medium gets heated.

How does the rate of heating depends on radius of body when it attains terminal velocity?

Ans. Rate of heat produced $\propto r^5$

Solution. Rate of heat produced $= F \times v$

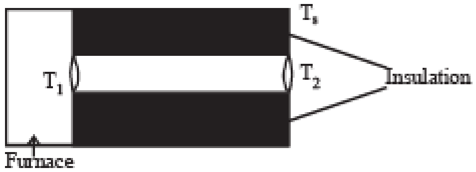
$= (6 \pi \eta r v) v$ [$\therefore$ Viscous $F = 6 \pi \eta r v$].

$$= (6 \pi \eta r)\left[\frac{2}{9}\frac{(\sigma - \rho) r^2 g}{\eta}\right]^2$$

$$\left[\because \text{Terminal velocity} = \frac{2}{9}\frac{(\sigma-\rho)r^2 g}{\eta}\right]$$

$\Rightarrow$ Rate of heat produced $\propto r^5$

Q.39. A cylindrical rod of length l, thermal conductivity K and area of cross section A has one end in the furnace at temperature T_1 and the other end in surrounding at temperature T_2. Surface of the rod exposed to the surrounding has emissivity e. Also $T_2 = T_s + \Delta T$ and $T_s >> \Delta T$. If $T_1 - T_s \propto \Delta T$, **find the proportionality constant.**



Ans. $\left[1 + \frac{4e\sigma \ell T_s^3}{K}\right]$

Solution.

From the figure it is clear that emission takes place from the surface at temperature T_2 (circular cross section). Heat conduction and radiation through lateral surface is zero.

Heat conducted through rod is

$$Q = \frac{KA(T_1 - T_2)\Delta t}{\ell}$$

Energy emitted by surface of rod in same time Δt, is

$$E = e\sigma A(T_2^4 - T_s^4)\Delta t$$

Since rod is at thermal equilibrium therefore E = Q

$$\text{hence, } \frac{KA(T_1 - T_2)\Delta t}{\ell} = e\sigma A(T_2^4 - T_s^4)\Delta t$$

$$\Rightarrow T_1 - T_2 = \frac{e\sigma(T_2^4 - T_s^4)\ell}{K}$$

Here $T_2 - T_s = \Delta T$ and $T_s >> \Delta T$

$$T_1 - (\Delta T + T_s) = \frac{e\sigma\ell}{K}\left[(\Delta T + T_s)^4 - T_s{}^4\right]$$

$$T_1 - (\Delta T + T_s) = \frac{e\sigma\ell}{K} \times T_s{}^4\left[\left(1 + \frac{\Delta T}{T_s}\right)^4 - 1\right]$$

$$T_1 - (\Delta T + T_s) = \frac{e\sigma\ell}{K} \times T_s{}^4\left[1 + \frac{4\Delta T}{T_s} - 1\right]$$

$$\text{or } T_1 - (T_s + \Delta T) = \frac{4e\sigma\,\ell}{K}T_s^3\Delta T$$

$$\text{or } T_1 - T_s = \left(\frac{4e\sigma\ell T_s^3}{K} + 1\right)\Delta T$$

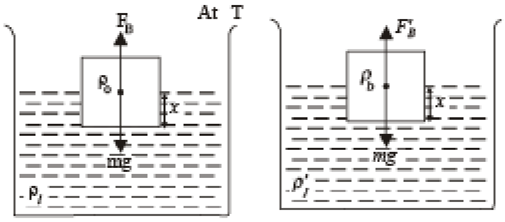
$$\therefore \text{ The proportionality constant} = \left(1 + \frac{4e\sigma\ell T_s^3}{K}\right)$$

Q.40.A cubical block of co-efficient of linear expansion as is submerged partially inside a liquid of co-efficient of volume expension γℓ. On increasing the temperature of the system by ΔT,the height of the cube inside the liquid remains unchanged. Find the relation between α_s and γℓ.

Ans. γ = 2α

Solution. Initially, at temperature T $F_B = mg$

$$Ax\rho_\ell g = AL\rho_b g$$



$$\Rightarrow \quad x\rho_\ell = L\rho_b \quad \text{...(i)}$$

At temperature $T + \Delta T$

$F_B' = mg$

$A'x\rho'_\ell g = AL\rho_b g$ [mg remains the same as above]

Now, $A' = A(1 + 2\alpha\Delta T)$

$\rho'_\ell = \rho_\ell(1 - \gamma\Delta T)$

$\therefore \quad A(1 + 2\alpha\,\Delta T)\,x\,\rho_\ell(1 - \gamma\Delta T)g = AL\rho_b g$

$\Rightarrow \quad x\rho_\ell(1 + 2\alpha\,\Delta T)(1 - \gamma\Delta T) = L\rho_b$

$\Rightarrow \quad x\rho_\ell(1 + 2\alpha\,\Delta T)(1 - \gamma\Delta T) = x\rho_\ell$ [From (i)]

$\Rightarrow \quad 1 + 2\alpha\,\Delta T - \gamma\,\Delta T = 1$

$\Rightarrow \quad \gamma = 2\alpha$

Q.41. A cylinder of mass 1 kg is given heat of 20,000J at atmospheric pressure. If initially the temperature of cylinder is 20°C, find

(a) final temperature of the cylinder. (b) work done by the cylinder. (c) change in internal energy of the cylinder

(Given that specific heat of cylinder = 400 J kg^{-1} $°C^{-1}$, coefficient of volume expansion = 9 × 10^{-5} $°C^{-1}$, Atmospheric pressure = 10^5 N/m^2 and Density of cylinder = 9000 kg/m^3)

Ans. 69.99°C, 0.0499J, 19999.95 J

Solution. (a) Heat supplied to the cylinder = Energy used to raise the temperature of cylinder + Energy used for work done by the cylinder.

Energy used to raise the temperature = $mc\Delta T = 1 \times 400 \times (T - 20)$ ... (i)

where T°C is the final temperature of the cylinder.

Energy used for work done = $P_{atm} (V_2 - V_1) = 10^5 (V_2 - V_1)$ ... (ii)

The final volume $V_2 = V_1 [1 + \gamma (T - 20)]$

$\Rightarrow V_2 - V_1 = V_1 \gamma (T - 20)$ ... (iii)

From (ii) and (iii),

Energy used for work done $= 10^5 V_1 \gamma (T - 20)$

$$= 10^5 \times \frac{1}{9000} \times 9 \times 10^{-5} (T - 20) \quad \left[\because V_1 = \frac{m}{d} = \frac{1}{9000}\right]$$

$$= 0.001 (T - 20) \quad \text{... (iv)}$$

$\therefore$ Heat supplied to the cylinder

$$= 400 (T - 20) + 0.001 (T - 20)$$

$$20{,}000 = 400.001 (T - 20)$$

$$\Rightarrow T = 69.99°C \approx 70°C$$

(b) Work done = 0.001 (69.99 – 20) = 0.0499 J

(c) Change in internal energy = 20,000 – 0.0499 = 19999.95 J.

Q.42. 0.05 kg steam at 373 K and 0.45 kg of ice at 253K are mixed in an insulated vessel. Find the equilibrium temperature of the mixture. Given, L_{fusion} = 80 cal/g = 336 J/g, $L_{vaporization}$ = 540 cal/g = 2268 J/g, S_{ice} = 2100 J/Kg K = 0.5 cal/gK and S_{water} = 4200 J/Kg K = 1 cal/gK

Ans. 273K or 0°C

Solution.

(1) Heat lost by steam at 100°C to change to 100°C water mL_{vap} = 0.05 × 2268 × 1000 = 1,13,400 J

(2) Heat lost by 100°C water to change to 0°C water = 0.05 × 4200 × 100 = 21,000 J

(3) Heat required by 0.45 kg of ice to change its temperature from 253 K to 273 K = $m \times S_{ice} \times \Delta T$ = 0.45 × 2100 × 20 = 18,900 J

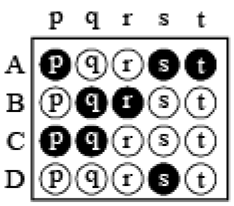
(4) Heat required by 0.45 kg ice at 273 K to convert into 0.45 kg water at 273 K = mL_{fusion} = 0.45 × 336 × 1000 = 151,200 J

From the above data it is clear that the amount of heat required by 0.45 kg of ice at 253 K to convert into 0.45 kg of water at 273 K (1,70,100 J) cannot be provided by heat lost by 0.05 kg of steam at 373 K to convert into water at 273 K.

Therefore the final temperature will be 273 K or 0°C.

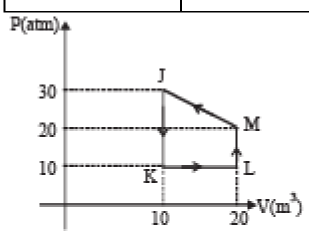
Match the Following

Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labelled A, B, C and D, while the statements in ColumnII are labelled p, q, r and s. Any given statement in Column-I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example : If the correct matches are A-p, s and t; B-q and r; C-p and q; and D-s then the correct darkening of bubbles will look like the given.



Q.1.Heat given to process is positive, match the following option of Column I with the corresponding option of column II :

(A) JK	(p) vW > 0
(B) KL	(q) $\Delta Q < 0$
(C) LM	(r) $\Delta W < 0$
(D) MJ	(s) $\Delta Q > 0$



Ans.(A)-(q); (B)-(p, s); (C)-(s); (D)-(q, r)

Solution.

(A) → (q) : JK is a isovolumic process. Therefore work done is zero. But there is decrease in pressure. Now $\Delta Q = \Delta U + \Delta W$. Therefore $\Delta Q = \Delta U$. In this case $\Delta U = nCv\Delta T$ and $P \mu T$.

Since pressure has decreased means temperature has decreased. Therefore ΔU is negative and so is ΔQ.

(B) → (p, s) : KL is a isobaric process. Pressure is constant.

The volume is increasing therefore $\Delta W > 0$. Also there is an increase in temperature. For both the case heat is absorbed.

Therefore $\Delta Q > 0$.

(C) → (s) : LM is a isovolumic process. Therefore work done is zero. The process is accompanied by increases in pressure. In this case, the temperature has increased and therefore $\Delta U > 0$. Therefore $\Delta Q > 0$.

(D)→ (q, r) : The process MJ is accompained with decrease in volume. Therefore $\Delta W < 0$. Also from the graph we can conclude that the temperature in the process decreases.

Therefore DU is also negative ⇒ $\Delta Q < 0$.

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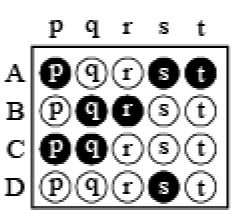
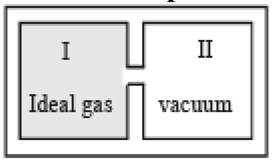
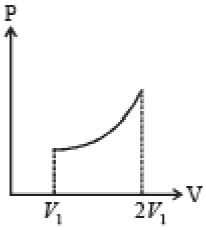


Q.2. Column I contains a list of processes involving expansion of an ideal gas. Match this with Column II describing the thermodynamic change during this

process. Indicate your answer by darkening the appropriate bubbles of the 4 × 4 matrix given in the ORS.

Column I	**Column II**
An insulated container has two chambers separated by a valve. Chamber I contains an ideal gas and the Chamber II has vacuum. The valve is opened. 	**(p) The temperature of the gas decreases**
An ideal monoatomic gas expands to twice its remains original volume such that its pressure $P \propto 1/V^2$ where V is the volume of the gas	**q) The temperature of the gas increases or constant**
An ideal monoatomic gas expands to twice its original volume such that its pressure $P \propto 1/V^{4/3}$	**(r) The gas loses heat where V is its volume**
An ideal monoatomic gas expands such that its pressure P and volume V follows the behaviour shown in the graph 	**(s) The gas gains heat**

Ans. (A)-(q); (B)-(p, r); (C)-(p, s); (D)-(q, s)

Solution. (A)-(q) : As the ideal gas expands in vacuum, no work is done (W = 0). Also the container is insulated therefore no heat is lost or gained (Q = 0). According to first law of thermodynamics $\Delta U = Q + W$ $\therefore \Delta U = 0$

⇒ There is no change in the temparature of the gas (B)-(p, r) : Given $PV^2 =$

constant ....(i)

Also for an ideal gas PV/T = constant

From (i) & (ii) $V \times T$ = constant As the gas expands its volume increases and temperature decreases

∴ (p) is the correct option To find whether heat is released or absorbed let us find a relationship between Q and change in temperature ΔT.

We know that $Q = n C \Delta T$ ...(i)

where C = molar specific heat Also for a polytropic process we have

$$C = C_v + \frac{R}{1-n} \quad \text{and} \quad PV^n = \text{constant}$$

Here PV^2 = Constant. Therefore n = 2

$$\therefore C = C_v + \frac{R}{1-2} = C_v - R$$

For monoatomic gas $C_v = \frac{3}{2}R$

$$\therefore C = \frac{3}{2}R - R = \frac{R}{2}$$

Substituting this value in (1) we get

$$Q = n \times \frac{R}{2} \times \Delta T.$$

In this case the temperature decreases i.e. ΔT is negative.

Therefore Q is negative. This in turn means that heat is lost by the gas during the process. (r) is the correct option. (C)-(p, s) : Proceeding in the same way we get in this case $V^{1/3} \times T$ = constant

⇒ As the gas expands and volume increases, the temperature decreases. Therefore

(p) is the correct option In this process, $x = \frac{4}{3}$.

$$\therefore C = C_v + \frac{R}{1-\frac{4}{3}} = \frac{3}{2}R + \frac{3R}{-1} = \frac{3}{2}R - 3R = \frac{-3R}{2}$$

$$\therefore Q = n\left(\frac{-3R}{2}\right)\Delta t$$

As ΔT is negative, Q is positive. This in turn means that heat is gained by the gas during the process (s) is the correct option.

(D)-(q, s) : Also $\Delta T = \frac{\Delta(PV)}{nR}$

Here Δ(PV) is positive ∴ ΔT is positive

∴ temperature increase s (q) is the correct option From the graph it is clear that during the process the pressure of the gas increases which shows that the internal energy of the gas has increased. Also the volume increases which means work is done by the system which needs energy.From these two interpretation we can comfortably conclude that the gas gains heat during the process. (s) is the correct option.

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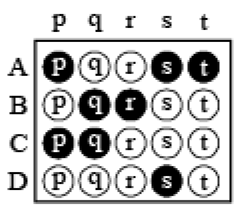


Q.3. One mole of a monatomic gas is taken through a cycle ABCDA as shown in the P-V diagram. Column II give the characteristics involved in the cycle. Match them with each of the processes given in Column I.

Column I	Column II
(A) Process A → B	(p) Internal energy decreases
(B) Process B → C	(q) Internal energy increases
(C) Process C → D	(r) Heat is lost
(D) Process D → A	(s) Heat is gained
	(t) Work is done on the gas

Ans. (A)-(p, r, t); (B)-(p, r); (C)-(q, s); (D)-(r, t)

Solution. (A) Process A→ B This is an isobaric process in which the volume of the gas decreases. Therefore work is done on the gas.

$W = P(3V - V) = 2PV$

Also temerature at B is less than temperature at A \ Heat is lost & internal energy decreases. (p, r, t) are correct matching

(B) Process B → C This is an isovolumic/isochoric process in which the pressure decreases Here temperature at B is less than temperature at C. \ Heat is lost and internal energy decreases. (p, r) are correct matching.

(C) Process C → D This is isobaric expansion where temperature at D is greater than temperature at C. Therefore internal energy increases and heat is gained. (q, s) are correct matching

(D) D → A

This is a process in which volume decreases. Therefore work is done on the gas.

Applying PV = nRT

$$\text{for D } P(9V) = 1\,RT_D \therefore T_D = \frac{9PV}{R}$$

$$\text{for A } 3P(3V) = 1RT_A \therefore T_A = \frac{9PV}{R}$$

$$\Rightarrow T_A = T_D \qquad \therefore \Delta U = 0$$

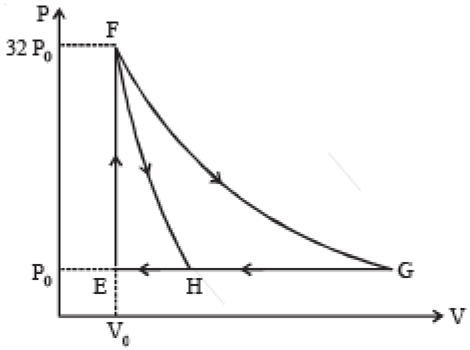
$$\text{Now, } \Delta Q = \Delta U + W \qquad \therefore \Delta Q = W.$$

The energy obtained by the gas by work done on it is lost to the surroundings as Δ U = 0.

∴ (r, t) are correct matching.

Following question has matching lists. The codes for the list have choices (a), (b), (c) and (d) out of which ONLY ONE is correct.

Q.4. One mole of a monatomic ideal gas is taken along two cyclic processes E → F → G v E and E → F v H v E as shown in the PV diagram. The processes involved are purely isochoric, isobaric, isothermal or adiabatic.



Match the paths in List I with the magnitudes of the work done in List II and select the correct answer using the codes given below the lists.

P. G → E	1. 160 P_0V_0 $\ln_2$
Q. G → H	2. 36 P_0V_0

R. F → H	3. 24 P_0V_0
S. F → G	4. 31 P_0V_0

Codes:

	P	Q	R	S
(a)	4	3	2	1
(b)	4	3	1	2
(c)	3	1	2	4
(d)	1	3	2	4

Ans. (a)

Solution.

$W_{GE} = P_0(V_0 - 32V_0) = -31P_0V_0$

$W_{GH} = P_0(8V_0 - 32V_0) = -24P_0V_0$

$$(W_{FH})_{adiabatic} = \frac{P_0(8V_0) - 32P_0(V_0)}{1 - \frac{5}{3}} = 36P_0V_0$$

$$(W_{FG})_{isothermal} = 1(32P_0V_0)\log_e \frac{32V_0}{V_0}$$

$= 32P_0V_0 \log_e 2^5$

$= 160P_0V_0 \log_e 2$

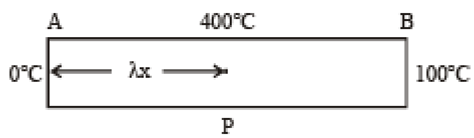
(a) is the correct option

Integer Value

Q.1.A metal rod AB of length 10x has its one end A in ice at 0.°C, and the other end B in water at 100 °C. If a point P onthe rod is maintained at 400 °C, then it is found that equal amounts of water and ice evaporate and melt per unit time. The latent heat of evaporation of water is 540 cal/g and latent heat of melting of ice is 80 cal/g. If the point P is at a distance of λx from the ice end A, find the value λ. [Neglect any heat loss to the surrounding.]

Ans.9

Solution.



For heat flow from P to 0

$$L_f \frac{dm_1}{dt} = \frac{KA\,400}{\lambda x} \quad ..... (i)$$

For heat flow from P to B

$$L_{vap} \frac{dm_2}{dt} = \frac{KA\,300}{10x - \lambda x} \quad ..... (ii) \left[\text{Given } \frac{dm_1}{dt} = \frac{dm_2}{dt}\right]$$

On solving (i) and (ii), we get **λ** = 9

Q.2. A piece of ice (heat capacity = 2100 J kg^{-1} °C^{-1} and latent heat = 3.36 × 10^5 J kg^{-1}) of mass m grams is at –5°C at atmospheric pressure. It is given 420 J of heat so that the ice starts melting.Finally when the ice-water mixture is in equilibrium, it is found that 1 gm of ice has melted. Assuming there is no other heat exchange in the process, the value of m is

Ans. 8

Solution. Heat supplied = Heat used in converting m grams of ice from –5°C to 0°C + Heat used in converting 1 gram of ice at 0°C to water at 0°

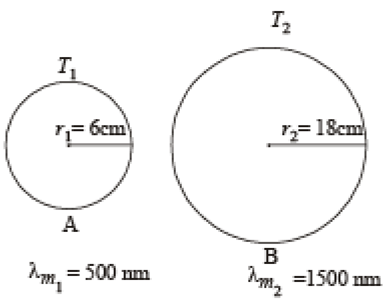
$$\Rightarrow 420 = m \times \frac{2100}{1000} \times 5 + \frac{1 \times 3.36 \times 10^5}{1000}$$

$$\Rightarrow 420 = m \times 10.5 + 336 \qquad \therefore m = \frac{84}{10.5} = 8 \text{ grams}$$

Q.3. Two spherical bodies A (radius 6 cm) and B(radius 18 cm) are at temperature T¹ and T², respectively. The maximum intensity in the emission spectrum of A is at 500 nm and in that of B is at 1500 nm. Considering them to be black bodies, what will be the ratio of the rate of total energy radiated by A to that of B?

Ans. 9

Solution.



$$\frac{\text{Rate of total energy radiated by A}}{\text{Rate of total energy radiated by B}}$$

$$= \frac{\sigma T_1^4 (4\pi r_1^2)}{\sigma T_2^4 (4\pi r_2^2)} = \left(\frac{T_1}{T_2}\right)^4 \times \left(\frac{r_1}{r_2}\right)^2$$

$$= \left(\frac{\lambda_{m_2}}{\lambda_{m_1}}\right)^4 \left(\frac{r_1}{r_2}\right)^2 \left[\because \frac{T_1}{T_2} = \frac{\lambda_{m_2}}{\lambda_{m_1}} \text{ by Wein 's law}\right]$$

$$= \left(\frac{1500}{500}\right)^4 \left(\frac{6}{18}\right)^2 = 9$$

Q.4. A diatomic ideal gas is compressed adiabatically to 1/32 of its initial volume. If the initial temperature of the gas is Ti (in Kelvin) and the final temperature is a T_i, the value of a is

Ans. 4

Solution. For an adiabatic process, the temperature-volume relationship is

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1} \Rightarrow T_1 = T_2\left(\frac{V_2}{V_1}\right)^{\gamma-1}$$

Here $\gamma = 1.4$ (for diatomic gas). $V_2 = \frac{V_1}{32}, T_1 = T_i, T_2 = aT_i$

$$\therefore T_i = aT_i\left[\frac{1}{32}\right]^{1.4-1} = aT_i\left[\frac{1}{2^5}\right]^{0.4} = \frac{aT_i}{4} \qquad \therefore a = 4$$

Q.5. Steel wire of lenght 'L' at 40°C is suspended from the ceiling and then a mass 'm' is hung from its free end. The wire is cooled down from 40°C to 30°C to regain its original length 'L'. The coefficient of linear thermal expansion of the steel is 10^{-5} /° C, Young's modulus of steel is 10^{11} N/m^2 and radius of the wire is 1 mm. Assume that L >>diameter of the wire. Then the value of 'm' in kg is nearly

Ans. 3

Solution. We know that

$$Y = \frac{mg/A}{\Delta\ell/\ell} = \frac{mg\ell}{A\Delta\ell} \qquad ....(1)$$

Also $\Delta\ell = \ell\,\alpha\,\Delta T$ ...(2)

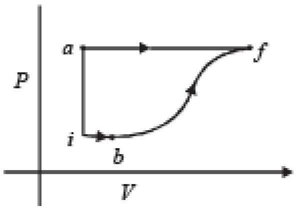
From (1) and (2)

$$Y = \frac{mg\ell}{A\ell\alpha\Delta T} = \frac{mg}{A\alpha\Delta T}$$

$$\therefore\ m = \frac{YA\alpha\Delta T}{g} = \frac{10^{11}\times\pi(10^{-3})^2\times10^{-5}\times10}{10} = \pi \approx 3$$

Q.6.A thermodynamic system is taken from an initial state i with internal energy U_i = 100 J to the final state f along two different paths iaf and ibf, as schematically shown in the figure. The work done by the system along the paths af, ib and bf are W_{af} = 200 J, W_{ib} = 50 J and W_{bf} =100 J respectively. The heat supplied to the system along the path iaf, ib and bf are Q_{iaf}, Q_{ib} and Q_{bf} respectively. If the internal energy of the system in the state b is

$U_b = 200$ J and $Q_{iaf} = 500$ J, The ratio $\frac{Q_{bf}}{Q_{ib}}$ is



Ans. 2

Solution. Applying first law of thermodynamics to path iaf

$$Q_{iaf} = \Delta U_{iaf} + W_{iaf}$$
$$500 = \Delta U_{iaf} + 200 \qquad \therefore \Delta U_{iaf} = 300\text{ J}$$

Now,

$$Q_{ibf} = \Delta U_{ibf} + W_{ib} + W_{bf}$$
$$= 300 + 50 + 100$$
$$Q_{ib} + Q_{bf} = 450\text{ J} \qquad \text{...(1)}$$

Also $Q_{ib} = \Delta U_{ib} + W_{ib}$

$$\therefore Q_{ib} = 100 + 50 = 150\text{ J} \qquad \text{...(2)}$$

From (1) & (2) $\dfrac{Q_{bf}}{Q_{ib}} = \dfrac{300}{150} = 2$

Q.7. Two spherical stars A and B emit blackbody radiation. The radius of A is 400 times that of B and A emits 104 times the power emitted from B. The ratio $\left(\frac{\lambda_A}{\lambda_B}\right)$ of their wavelengths λ_A and λ_B at which the peaks occur in their respective radiation curves is

Ans.2

Solution. $\dfrac{P_A}{P_B} = \dfrac{A_A}{A_B}\dfrac{T_A^4}{T_B^4} = \dfrac{A_A}{A_B} \times \dfrac{\lambda_B^4}{\lambda_A^4}$

$$\therefore \frac{\lambda_A}{\lambda_B} = \left[\frac{A_A}{A_B} \times \frac{P_B}{P_A}\right]^{\frac{1}{4}} = \left[\frac{R_A^2}{R_B^2} \times \frac{P_B}{P_A}\right]^{\frac{1}{4}} = \left[\frac{400 \times 400}{10^4}\right]^{\frac{1}{4}}$$

$$\therefore \frac{\lambda_A}{\lambda_B} = 2$$

Q.8. A metal is heated in a furnace where a sensor is kept above the metal surface to read the power radiated (P) by the metal.The sensor has a scale that displays $\log_2$, (P/P_0), where P_0 is a constant. When the metal surface is at a temperature of 487°C, the sensor shows a value 1. Assume that the emissivity of the metallic surface remains constant. What is the value displayed by the sensor when the temperature of the metal surface is raised to 2767°C?

Ans. 9

Solution.

Here $P \propto T^4$ or $P = P_0 T^4$

$$\therefore \log_2 P = \log_2 P_0 + \log_2 T^4 \quad \therefore \log_2 \frac{P}{P_0} = 4 \log_2 T$$

$$\text{At T} = 487°\text{C} = 760\,\text{K}, \log_2 \frac{P}{P_0} = 4 \log_2 760 = 1 \ldots (1)$$

At T = 2767°C = 3040K,

$$\log_e \frac{P}{P_0} = 4 \log_2 3040 = 4 \log_2 (760 \times 4)$$

$= [4 \log_2 760 + \log_2 2^2]$

$= 4\log_2 760 + 8 = 1 + 8 = 9$